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A heuristic algorithm for the Capacitated Vehicle Routing Problem with Synchronized Pick-ups and Drop-offs: A case study for medications delivery and supervision in DR Congo

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A heuristic algorithm for the Capacitated Vehicle Routing Problem with Synchronized Pick-ups and Drop-offs: A case study for medications delivery and supervision in DR Congo

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RÉSUMÉ

Dans des contextes de post-urgence tels que le vit la partie occidentale de la République Démocratique du Congo (RDC), l'un des défis cruciaux auxquels font face les hôpitaux ruraux est de maintenir un niveau de médicaments essentiels dans la pharmacie. Sans ces médicaments pour traiter les maladies graves, l'impact sur la santé de la population est significatif. Les hôpitaux encourent également des pertes financières dues à la péremption lorsque trop de médicaments sont commandés. De plus, les coûts du transport des médicaments ainsi que du superviseur sont très élevés pour les hôpitaux isolés ; les coûts du transport peuvent à eux seuls dépasser ceux des médicaments. En utilisant la province du Bandundu, RDC pour une étude de cas, notre recherche tente de déterminer la faisabilité (en termes et de la complexité du problème et des économies potentielles) d'un problème de routage synchronisé pour la livraison de médicaments et pour les visites de supervision. Nous proposons une formulation du problème de tournées de véhicules avec capacité limitée qui gère plusieurs exigences nouvelles, soit la synchronisation des activités, la préséance et deux fréquences d'activités. Nous mettons en œuvre une heuristique « cluster first, route second » avec une base de données géospatiales qui permet de résoudre le problème. Nous présentons également un outil Internet qui permet de visualiser les solutions sur des cartes. Les résultats préliminaires de notre étude suggèrent qu'une solution synchronisée pourrait offrir la possibilité aux hôpitaux ruraux d'augmenter l'accessibilité des services médicaux aux populations rurales avec une augmentation modique du coût de transport actuel.

Mots clés: CVRP, synchronisation, GIS, voronoi, logistique post-urgence, précédence

ABSTRACT

In post-emergency contexts such as in Western Democratic Republic of the Congo, also known as DR Congo, one of the crucial challenges that rural hospitals face is maintaining a pharmacy with essential medications and supplies. There is significant negative humanitarian impact when hospitals do not have essential medications for treatable life-threatening diseases; hospitals also incur financial losses when too much medication is ordered and it expires. Moreover, the cost of transporting medications and providing on-site supervision to remote hospitals is an extremely expensive endeavour. In some cases the transportation costs alone can surpass the cost of the medications. Using the province of Bandundu, DR Congo as a case study, we attempt to determine the feasibility (in terms of both problem complexity and potential savings) of a synchronized routing problem for medication delivery and on-site supervision visits. We propose a Capacitated Vehicle Routing Problem formulation that handles several novel requirements including activity-wise synchronization, precedence, and two activity frequencies. We implement a novel cluster-first, route-second heuristic with a geospatially-enabled database to solve the problem. We also present a Web-based tool to visualize the solutions in a map. The preliminary results of the research suggest that a synchronized solution could allow rural hospitals to increase the accessibility of medical services to rural populations with only a modest increase in transportation costs.

Keywords: CVRP, synchronization, GIS, Voronoi, post-emergency logistics, precedence

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LIST OF ABBREVIATIONS

VRP	Vehicle Routing Problem
CVRP	Capacitated Vehicle Routing Problem
DARP	Dial-A-Ride Problem
SP	Set Partitioning
MIP	Mixed Integer Problem
SQL	Structured Query Language
NGO	Non-Governmental Organization

To the dedicated healthcare practitioners who strive to improve healthcare in the DR Congo

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PREFACE

From 2009 to 2012, I worked in the DR Congo as co-coordinator of Mennonite Central Committee's Health Program, Menno-Santé. I had the privilege of working with officials from the DR Congo's ministry of public health, international non-governmental organizations (NGOs), churches, doctors, nurses, and hospital administrators on a program aimed at improving access to essential medications in four rural hospitals in the provinces of Bandundu and Kasai Occidentale. Based on that experience, I gained some insight into the logistical challenges that local health practitioners face in their efforts to provide better healthcare to the local population. Given my academic background in operations management, I naturally became interested in these logistical challenges, particularly the questions pertaining to medication inventory management. I hope the results of this thesis work will assist these practitioners in their work.

CHAPTER 1

INTRODUCTION

This chapter briefly introduces the transportation context in Bandundu, DR Congo, the vehicle routing problem, the research scope and objectives, and the overall organization of the thesis.

1.1 Transportation context in Bandundu, DR Congo

The present research aims to improve pharmacy management and hospital sustainability through improved medication delivery and supervision, in order to contribute to the humanitarian objective of improved access to medical care. The specific transportation context for our case study is the province of Bandundu, DR Congo. Since 1975, the DR Congo Ministry of Health [22], in collaboration with churches and NGOs, has progressively partitioned the country into health zones in a strategic plan to improve primary health care. In 2003 the number of zones increased from 300 to 515 [29] and has stayed at approximately this number since then. Given the total population of about 67.5 million [38], each health zone services 130,000 people on average. Typically, each health zone includes a single general reference hospital, secondary hospitals, clinics and health posts that employ a structured referral system. Figure 1.1 shows a map of the health zones and the estimated location of each general reference hospital. [1] Administrators of general reference hospitals and the health zones work together and sometimes in collaboration with church groups and NGOs to coordinate health services for the health zone's population.

This research focuses on delivering medication and supervising hospitals in the western province of Bandundu. In 2008, Bandundu had a population of 5.5 million [35]. The province is divided into 52 administrative health zones serving about 106,000 people each. We are primarily concerned with the general reference hospitals, from now on re-

1. We describe in detail how the estimated hospital coordinates were obtained in Section I.

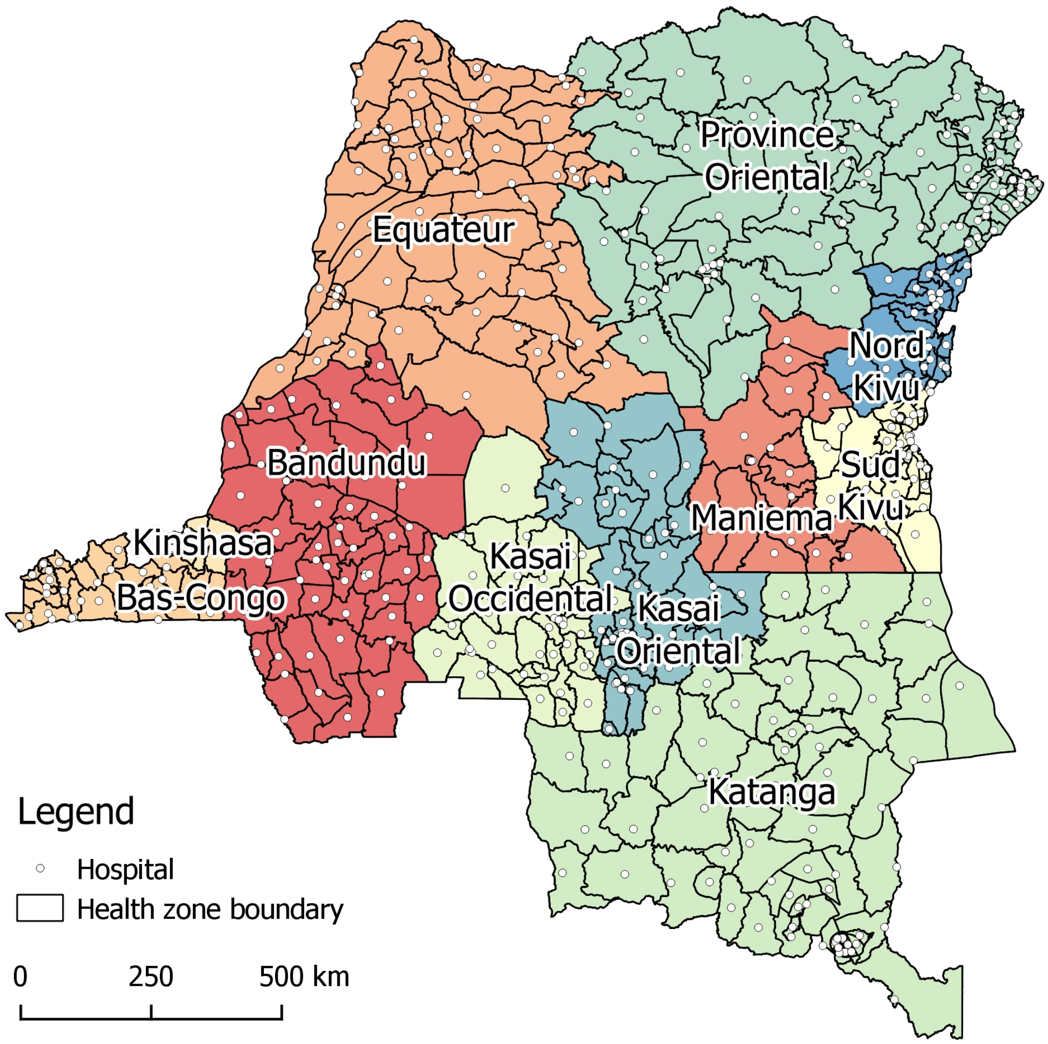


Figure 1.1: Provincial boundaries, health zones, and estimated general reference hospital locations in the DR Congo

ferred to simply as hospitals. We chose to focus on the hospitals in part due to the scope of our research, but also because in our experience the general reference hospital has a significant impact on the entire health zone functionality with both direct and indirect benefits to the population. Moreover, the hospital administration often overlaps with the health zone administration, so that learnings from logistics can ostensibly be shared and extended to have an even greater impact.

1.2 Introduction to the problem

Importance and challenges of pharmacy management

One of the crucial challenges of a rural hospital is to maintain a pharmacy with essential medications and supplies as required by the Ministry of Public Health. It is easy to imagine the negative humanitarian impact of not having essential medications for treatable life-threatening diseases such as malaria, diarrhoea, and pneumonia as well as the financial losses that occur when too much medication is ordered and it expires. Not having the medical supplies and medications required for surgery leads to decreased perceived credibility for the hospital and insufficient revenue needed to support hospital staff.

In his doctoral research on hospitals in Cameroon and DR Congo, Sthreshley [33] argues that pharmacy management is the second-most significant factor in hospital financial sustainability, with the most significant being human resource management. In other words, Sthreshley claims that if one wants to increase the autonomy of a hospital, efforts to improve human resource management and pharmacy management will likely have the most impact. The economic impact of poor pharmacy management is immediately evident when one considers lost opportunity cost, logistical inefficiencies, and the large proportion of hospital financial resources expended on medications. For hospital staff and administrators, increased hospital autonomy translates immediately into better pay, improved job stability, and improved training potential. In addition, savings in medication transportation can also be passed on to the rural population via reduced medication prices, increased availability of medical supplies, and eventually more affordable and higher quality medical care.

Relevance of transportation to pharmacy management

As we worked alongside hospital administrators and regional coordinators in DR Congo, it was clear that they faced many difficult challenges when addressing the issue of pharmacy management. However, two particular challenges captured our attention. The first was finding reliable transport options to deliver medication to remote regions where road infrastructure is minimal and transport costs, both by air and by ground, are extremely high. A second important and often overlooked challenge was scheduling reliable supervision visits for the purpose of training and follow-up regarding ongoing medication procurement planning.

The cost of delivering medication by air or ground is exorbitant, both financially, due to the high cost of transport, and in terms of travel time for ground transport. Currently, hospital managers generally use ground transport primarily because ground transport can be procured on an ad-hock basis. These costs are passed on to the population in the form of high medication costs and lower availability. The cost of transporting medications to some of the hospitals we worked with was almost as much as the cost of the medications at depot prices. Furthermore, the frequency of medication orders is often uncertain; there can be periods of five to six months between orders. Hospital administrators typically order on an ad hoc basis, both because planning for and financing large orders from the depot were beyond the hospital's administrative capacity, and because finding vehicles to deliver to rural hospitals is very difficult.

Menno-Santé coordinators, our colleagues, local coordinators and hospital administrators introduced a plan to have the hospitals make a large enough order to fill an entire vehicle, typically about 1000 kg of medications. The target order interval was about four months. However, this approach also had its challenges: the large interval between orders resulted in less frequent supervision and increased uncertainty with respect to inventory levels, resulting in stock outages and loss of expired medications. The idea of regular monthly orders was difficult to imagine because individual hospital demand was not sufficient to fill the aircraft we used.

Planning predictable pharmacy supervision/training of hospital administrators is a second important challenge. Hospitals and health zones are supervised by district and zone coordinators. However, our three years experience of managing the Menno-Santé health project led us to conclude that the type, level, and frequency of supervision usually available was not sufficient to truly empower local hospital administrators to improve pharmacy management. We found that in addition to rationally stocked medical supplies, frequent training and supervision was a key factor in obtaining lasting impact. Transporting supervisors to hospitals by air would ideally require two return trips — one trip to drop off the supervisor and one trip to pick him/her up after a few days. However, this is far too expensive. Travelling by ground is possible, but incurs a high opportunity cost due to the time required for supervisors to reach remote hospitals. It was evident that there was immediate benefit to combining the supervision trips with the medication deliveries by air transport, but it was not easy to find a routing solution even for three hospitals.

The Menno-Santé coordinators did their best to find an optimal medication routing policy, but it became clear that this problem would require a decision tool to handle the complexity.

1.3 Research scope and objectives

Since medication delivery and supervision routing represent such a high proportion of the total cost in providing medication to rural hospitals, and because they present us with an opportunity to make a significant impact on overall hospital sustainability, we chose to address the air transportation problem in this thesis.

The scope and objectives of our research evolved as we considered the following questions: Would it be possible to a) consider monthly delivery to each hospital by preselecting clusters of hospitals and having the clusters share the cost of a single route, and b) combine the monthly medication delivery trips with bi-annual supervision in such a way as to reduce overall costs for each hospital? If so, how much savings would we achieve (if any) and could we solve the problem in a reasonable amount of time?

Using real-world input and contextual data from the province of Bandundu, the objectives of the present research are to 1) present a new class of synchronized CVRP called the Capacitated Vehicle Routing Problem with Synchronized Pick-ups and Drop-offs (CVRP-SyncPD); [2] 2) implement a fully parameterized and iterative solving procedure using a geospatially-enabled database and a commercial solver; 3) compare the cost of monthly medication delivery solutions using our algorithm with existing strategies that deliver medications every four months; 4) implement an interactive visualization framework to display solutions for the province of Bandundu; and 5) provide a proof-of-concept for administrators interested in implementing a generic software application for use by humanitarian practitioners in the field.

1.4 Thesis organization

This thesis is organized into six chapters. In Chapter 1 we described the transportation context in the DR Congo, introduced the vehicle routing problem, and defined our research scope and objectives. In Chapter 2, we review the existing literature. In Chapter 3, we describe the problem instance in more detail and propose a mathematical model. In Chapter 4, we describe the heuristic algorithm used to solve an instance of the problem and the specific implementation used. In Chapter 5, we present the results for the problem instance. Finally, in Chapter 6 we review the contributions of this thesis and propose additional applications and further research.

2. We chose this abbreviation in order to avoid confusion with the existing Capacitated Vehicle Routing Problem with Synchronized Pick-up and Delivery (CVRPSPD) proposed by Subramanian et al. [34].

CHAPTER 2

LITERATURE REVIEW

The problem we are considering is a particular instance of the Capacitated Vehicle Routing problem (CVRP). We are attempting to find an optimal routing of aircraft that will satisfy the hospital medication demand and provide multi-day supervisory visits to each hospital. The CVRP is a generalization of the Travelling Salesman Problem (TSP). It attempts to minimize the routing cost given a set of clients with product demand while not exceeding the vehicle capacities and other constraints such as distance. The CVRP is interesting both from a theoretical point of view because it is an NP-Hard combinatorial optimization problem, and from a practical perspective because there are a wide range of commercial and humanitarian applications that can benefit from decision support tools based on the VRP. In the present research we are specifically interested in solution methodologies for the CVRP with special synchronization pick-up/drop-off properties applied in a context of humanitarian aid. In this literature review we highlight the lack of literature focused on humanitarian logistics during the reconstruction phase and we demonstrate how our problem, while apparently quite similar to existing CVRPs, has unique properties that have not yet been fully explored.

2.1 Literature on humanitarian logistics

Numerous publications argue that there is a lack of research on humanitarian logistics and we cite four of them here. Martinez et al. [26] argue that academic knowledge about fleet management in humanitarian operations is scarce. Van Wassenhove and Martinez [36] suggest that applying Operations Research to humanitarian supply chains can achieve significant improvements. Crum et al. [11] argue that supply chain design in humanitarian aid contexts would benefit from research with a perspective on sustainable, long-term development. Moreover, Kuntz [21] argues that there is a growing trend of literature written about humanitarian logistics. However, in his survey, Kuntz finds that no articles on humanitarian logistics were written specifically about the reconstruction stage. This is in contrast to 56% of the articles in his study that were dedicated to the (emergency) response stage. In the case of countries such as the DR Congo that have suffered from decades of civil conflict, the emergency aid response tends to be scaled back after the immediate emergency is over and humanitarian efforts are transitioned into longer-term reconstruction projects. The medication delivery and supervisor pick-up/drop-off problem that we are considering lies in that space of the literature that addresses logistics in this reconstruction phase.

2.2 CVRP literature

Much research has been done on the key features of the CVRP that we will develop in this paper: synchronization, reverse logistics, temporal precedence, and pick-ups/drop-offs have been amply covered in the literature. However, there are unique attributes of our problem that prevent existing formulations and solution methodologies from being directly applied. In this section we review the literature related to these unique attributes both in terms of problem formulation and solution methodology.

The existing literature does not provide a suitable formulation that could be used to describe the CVRP-SyncPD. This is due to several unique characteristics of our problem: its two task frequencies, its task-wise synchronization, its precedence requirements, and combinations of the above.

First, the problem has two routing delivery frequencies that we must solve simultaneously: the first frequency is tied to the monthly medication delivery requirements, while the second is tied to the bi-annual (once every six months) supervision pick-up and drop-off requirements.

Second, the problem has unique synchronization constraints. We must synchronize two tasks (medication and supervisor movement) with a single route. In his survey of VRPs with multiple synchronization constraints (VRPMS), Drexl [13] defines a VRPMS as ‘a vehicle routing problem where more than one vehicle may or must be used to fulfil a task’ and thus excludes the possibility of addressing the synchronization of several tasks to a single vehicle route. Moreover, none of the the synchronization categories he presents handle problems with multiple frequencies.

Third, the problem has pairwise precedence requirements with respect to medication deliveries: each hospital expects to receive its monthly medication order at approximately the same time each month. Bredströmand and Rönnqvist [9] present a general mathematical model for the combined vehicle routing and scheduling problem with time windows, pairwise temporal precedence, and pairwise synchronization between customer visits. Unfortunately, their formulation does not address the multiple-task-to-single-route synchronization required by our problem. Moreover, it requires a predefined pairwise precedence matrix as an input, whereas our problem does not specify a particular precedence for the medication delivery.

Fourth, various reverse logistics formulations such as the Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRP-SPD) proposed by Dethloff [12] handle CVRP problems with flow of goods to and from the depot, but do not handle the pick-up and drop-off requirements of our problem – notably that the supervisor must be able to travel from client to client, not just client back to depot. Finally, the Set Partitioning (SP) formulation of the CVRP first proposed by Balinski and Quant [7] and more recently re-iterated in Baldacci's exact method [5] does not handle the synchronization and pick-up/drop-off requirements of our problem. Although it is not a perfect fit for our problem, we note that the SP formulation was instrumental, both conceptually and technically, in the development of our formulation and eventual heuristic implementation. We will draw on the SP formulation to a large degree and extend it to handle the novel attributes of our problem.

As there was no previously existing formulation that could easily handle the unique properties of our problem, it naturally followed that there was no solution methodology for such a formulation either. However, there were several publications that provided conceptual clues for our eventual implementation. In particular, the cluster-first, route-second methods outlined by Laporte et al. [23], such as the sweep algorithm by Gillett and Miller [19], and the petal algorithm originally proposed by Foster [17], and extended by Ryan [31] and Renaud [30], propose algorithms based on radial ordering and clustering. Unfortunately, these methods could not handle the complex route attributes and route constraints that our problem requires. For instance, our route definition includes both delivery and pick-up/drop-off attributes, but the petal methods handle only the delivery requirements attribute. Moreover, the existing cluster-first, route-second methods could not handle an arbitrary number of route constraints in addition to the traditional maximum capacity constraint. It should be noted that although these methods did not apply directly to our problem, the concepts of ordering hospitals and clusters of hospitals radially, and the cluster-first, route-second method were used extensively in our implementation.

2.3 Conclusion

In this chapter, we have shown how the problem, to our knowledge, falls into the subset of literature on humanitarian logistics involved with reconstruction contexts that has not been studied specifically, and we have argued that the current CVRP literature does not apply to all aspects of our problem – specifically its two task frequencies, task-wise synchronization, precedence requirements, and/or combinations of these aspects. In the next chapter we describe the problem in detail, present a mathematical model formulation for it, and review its complexity.

CHAPTER 3

THE PROBLEM

In this chapter, we describe the Capacitated Vehicle Routing Problem with Synchronized Pick-ups and Drop-offs (CVRP-SyncPD) in the context of Bandundu, and we formulate the problem as a Mixed Integer Program (MIP) by extending the Set Partitioning formulation of the CVRP.

3.1 Conceptualizing the CVRP-SyncPD as a combined delivery and pick-up/drop-off problem

The CVRP-SyncPD was constructed to solve the medication delivery and supervisor pick-up/drop-off problems simultaneously for a single depot and a single supervisor. In this section, we outline the essence of the two base problems and the combined problem.

The medication delivery problem

The medication delivery problem consists of finding a feasible routing, given factors such as aircraft capacities (payload in kilograms), per-hospital monthly demand (medications in kilograms), hospital locations, and the location of a single depot. A valid routing solution must deliver medications to each hospital exactly once a month according to its monthly demand while not exceeding the capacity of the available aircraft. For simplicity, we do not consider stochastic aspects of the problem. The medication delivery problem can be solved in a reasonable amount of time using one of the many heuristic or exact CVRP algorithms available (see Laporte et al. [24], Semet et al. [32], and Poggi et al. [27] for recent surveys).

The supervision pick-up and drop-off problem

The supervision pick-up and drop-off problem consists of finding a feasible multi-month routing for one or more supervisors who must visit all the hospitals at least once every six months. We refer to this six-month period as the problem horizon. The supervisor starts and finishes at the depot and must visit a subset of the hospitals each month. [1] A unique aspect of this problem is that the supervisory visit lasts for a few days and the vehicle cannot wait on-site during the visit for various security and economic reasons. Consequently, a supervisor cannot simply tag along on existing medication delivery routes. Rather s/he must be dropped off and picked up on a subsequent route. This problem could be formulated in several ways, but the Dial-A-Ride Problem (DARP) formulation (see Cordeau and Laporte [10] for a recent survey) would be a good fit as it specifically addresses the pick-up and drop-off nature of the supervision problem and can handle multiple supervisors with time windows if necessary. For a single supervisor without time windows, it could also be formulated as a Travelling Salesman Problem (TSP).

The delivery and pick-up/drop-off problem

Synchronizing the medication delivery and supervision pick-up and drop-off tasks has the potential to reduce overall transportation costs significantly because we can share route segments. We synchronize the two tasks by constructing routes that deliver medications to a cluster of hospitals every month (rather than just one hospital every four months) and pick up and drop off the supervisor. Solutions to the combined problem ensure that a) medications are delivered at the same time each month, and b) the supervisor visits all hospitals at least once over the six-month horizon. As with the standard Set Partition formulation of a CVRP [7], the key to finding the optimal solution in a CVRP-SyncPD is determining an efficient method to find the optimal route set amongst the entire set of feasible routes.

1. This is not a management requirement of the problem per se, but generally speaking, it is a good practice to allow for periodic breaks for the supervisor. We use it in our model primarily to simplify the formulation.

3.2 Mathematical model

We now present the mathematical model for the CVRP-SyncPD which is an extension of the Set Partitioning formulation of the CVRP originally proposed by Balinski and Quandt [7] and referenced more recently in the context of an exact algorithm by Baldacci and Christofides [5]. In Section 3.2.1 we begin with a description of how we construct the routes and evaluate the route costs for the model. In Section 3.2.2 we formulate the problem as a MIP.

3.2.1 Constructing the routes and route costs

In our model, in addition to specifying which hospitals have medication deliveries, the routes also indicate where the supervisor is picked up and dropped off and the sequence in which delivery and supervisor pick-up/drop-off activities occur. Generating routes and route costs for the model can be summarized in the following four steps: **Step 1)** Generate a set of medication clusters, **Step 2)** Construct medication-supervision clusters associated with each medication cluster, **Step 3)** Construct routes associated with each medication-supervision cluster, and **Step 4)** Determine route costs. We describe these steps next.

Step 1: Generate a set of medication clusters

Let V be the set of hospitals and U be the set of feasible medication clusters. A feasible **medication cluster** $\mathscr{U} \in U$ is a subset of hospitals $i \in V$ for which the total medication demand does not exceed the capacity of the largest available aircraft. In the case of Bandundu, this capacity restriction limits the number of nodes in a medication cluster to a maximum of four hospitals. Consequently, we construct U by generating medication clusters for all combinations of up to four hospitals (including an empty medication cluster).

Step 2: Construct medication-supervision clusters associated with each medication cluster

The next step in the route generation process is to generate feasible medication-supervision clusters associated with each medication cluster. A **medication-supervision cluster** specifies which hospitals (or depot) have pick-ups and drop-offs in addition to the medication deliveries already specified in the medication cluster. It is a set of nodes containing the medication cluster, the depot, and up to two additional hospitals (for supervisor pick-up and/or drop-off). It does not include information about the order in which the nodes are visited (we determine the sequence in Step 3). Feasible medication-supervision clusters have the following two requirements: a) every hospital in the cluster must be associated with a medication delivery or pick-up/drop-off, or a combination of the two and b) there must be exactly one node (hospital or depot) associated with a supervisor pick-up and exactly one hospital (or the depot) associated with a supervisor drop-off. For each medication cluster and for each valid permutation of pick-up/drop-off nodes (only permutations that generate feasible medication-clusters are considered) we generate a feasible medication-supervision cluster. We describe the process of constructing medication-supervision clusters for each $\mathscr{U} \subset U$ in steps 2a, 2b, and 2c. We use the following route convention to describe the clusters and routes: node numbers followed by a P, D, and/or M stand for supervisor pick-up, drop-off, and medication delivery, respectively. The depot node is indicated with an index of 0. If a hospital (or depot) is prefixed by both P and D, this indicates that the supervisor is not moved during the route.

Step 2a: Generate medication-supervision clusters from medication-cluster $\mathscr{U}$

We consider the medication cluster $\mathscr{U} \cup \{0\}$ and generate a medication-supervision cluster associated with each feasible pick-up/drop-off permutation. For example, if we start with $\mathscr{U} = \{21, 25\}$ we would generate the following medication-supervision clusters:

medication-supervision clusters
{0PD, 21M, 25M}
{0P, 21DM, 25M}
{0P, 21M, 25DM}
{0, 21PM, 25DM}
{0D, 25M, 21PM}
{0, 21PDM, 25M}
{0, 25PM, 21DM}
{0D, 21PM, 25M}
{0, 21M, 25PDM}

Table 3.I: Sample medication-supervision clusters generated in Step 2a (no additional hospitals added)

There are two clusters of interest in Table 3.I. First, the cluster {0PD, 21M, 25M} will eventually result in a route called a **medication-only route** because the supervisor stays at the depot and the aircraft only delivers medication. Second, the cluster {0, 21PDM, 25M} indicates that the supervisor will remain at hospital 21. This kind of cluster produces routes that allow the supervisor to stay at a hospital during a particular travel day.

Step 2b: Add one pick-up/drop-off hospital to medication-cluster $\mathscr{U}$ and generate the associated medication-supervision clusters

In steps 2b and 2c, we add one and two additional hospitals (respectively) to each medication cluster in order to allow for routes that have a pick-up or drop-off at a hospital that is not already included in the medication cluster.

For all hospitals $i \in V \setminus \mathscr{U}$, we consider the set $\mathscr{U} \cup \{0, i\}$ and generate medication-supervision clusters for each feasible pick-up/drop-off permutation. For example, if we start with $\mathscr{U} = \{21, 25\}$, and consider one of the hospitals from $V \setminus \mathscr{U}$ (say 30), we would generate a subset of medication-supervision clusters as follows:

medication-supervision clusters
{0P, 21M, 25M, 30D}
{0D, 25M, 21M, 30P}
{0, 25PM, 21M, 30D}
{0, 25M, 21PM, 30D}
{0, 30P, 25DM, 21M}
{0, 30P, 25M, 21DM}
{0, 21M, 25M, 30PD}

Table 3.II: Sample medication-supervision clusters generated in Step 2b (one additional hospital added)

Medication-supervision clusters in which there are no medication deliveries and where the supervisor is moved to or from the depot are generated from the empty medication cluster $\mathscr{U} = \{\emptyset\}$. For example, the medication-supervision cluster {0D, 30P} would eventually become a route in which the supervisor is picked up from hospital 30 and dropped off at the depot. We need these kinds of routes to bring the supervisor back to the depot after the last medication delivery in a month.

Step 2c: Add two pick-up/drop-off hospitals to medication-cluster $\mathscr{U}$ and generate the associated medication-supervision clusters

For all hospital pairs $\{i, j\} \in V \setminus \mathscr{U}, i \neq j$, consider the set $\mathscr{U} \cup \{0, i, j\}$ and generate medication-supervision clusters for each feasible pick-up/drop-off permutation. For example, if we started with $\mathscr{U} = \{21, 25\}$ and considered one hospital pair $\{30, 40\}$, we would generate the following two medication-supervision clusters:

medication-supervision clusters
{0, 21M, 25M, 30P, 40D}
{0, 21M, 25M, 30D, 40P}

Table 3.III: Sample medication-supervision clusters generated in Step 2c (two additional hospitals added)

Medication-supervision clusters in which there are no medication deliveries and where the supervisor is moved from one hospital to another are generated from the empty medication cluster $\mathscr{U} = \{\emptyset\}$. For example, the medication-supervision cluster {0, 30P, 40D} would eventually generate a route in which the supervisor is picked up from hospital 30 and dropped off at hospital 40. Typically, we might use this kind of route to bring the supervisor to a different hospital after all the medication deliveries for a given month are complete.

Step 3: Construct routes for each medication-supervision cluster

In Step 3, we construct a single route for each medication-supervision cluster by determining the hospital visitation sequence. An initial sequence is determined by solving a Travelling Salesman Problem (TSP) on the nodes in the cluster to obtain the minimum route distance. If necessary, we reverse the initial sequence to ensure that the pick-up node never comes after the drop-off node. For example, if we take the medication-supervision cluster {0P, 21DM, 25M}, the TSP might give the sequence (0P, 25M, 21DM). We do not need to reverse the sequence because the pick-up node 0P, already precedes the drop-off hospital 21DM. Therefore, the following route will be generated: (0P – 25M – 21DM – 0).

In the case of a medication-supervision cluster that has a hospital that is assigned as both pick-up and drop-off but with no medication delivery, we do not consider that hospital when solving the TSP. Technically, it remains part of the route, but it is only present to allow the model to handle routes in which the supervisor remains at the hospital. For example: The medication-supervision cluster {0, 21M, 25M, 30PD} specifies that the supervisor remains at hospital 30 and the aircraft will deliver medications to hospitals 21 and 25. As such, the aircraft has no need to visit hospital 30. Therefore, we compute a solution to the TSP including only hospitals 21 and 25 and visually indicate that hospital 30 is not included in the actual route sequence by placing it in square parentheses: (0 – 25M – 21M – 0 [30PD]).

Step 4: Evaluating route costs

The route cost is determined as a function of total route distance and the aircraft cost per kilometre. The choice of aircraft is determined solely on the basis of the aircraft's capacity and its relative cost with respect to other aircraft costs per kilometre. For each route generated in Step 3, we generate its cost as follows:

1. Select the aircraft with the cheapest cost per kilometre and with the capacity sufficient to transport the total medication demand for all hospitals in the route.
2. Determine the route cost by multiplying the aircraft's cost per kilometre by the total distance of the route determined by the TSP solution in Step 3.

We have described how to generate the routes and route costs for the model and provided notation used to represent the routes in the model. In the next section we present the model as an MIP.

3.2.2 The CVRP-SyncPD model as an MIP

Let $V^* = V \cup \{0\}$ be the set of hospitals and depot node, R be the set of routes, and a_{ir}, p_{ir} and d_{ir} be binary coefficients that define the delivery, pick-up, and drop-off attributes of each route $r \in R$ as follows:

$$a_{ir} = \begin{cases} 1, & \text{if route } r \text{ delivers medication to hospital } i \in V \\ 0, & \text{otherwise} \end{cases}$$

$$p_{ir} = \begin{cases} 1, & \text{if route } r \text{ picks up supervisor at node } i \in V^* \\ 0, & \text{otherwise} \end{cases}$$

$$d_{ir} = \begin{cases} 1, & \text{if route } r \text{ drops off supervisor at node } i \in V^* \\ 0, & \text{otherwise} \end{cases}$$

Let c_r be the route cost for each route.

Let $K = \{1, 2, ..., 6\}$ be the set of **months** corresponding to each month in the problem horizon. Let $T = \{1, 2, ..., 12\}$ be the ordered set of **travel days** in a given month. Travel days do not usually correspond to consecutive calendar days as there must be two days between trips to satisfy the supervision requirements. Travel days allow the model to ensure the medication delivery order occurs on the same day every month and to ensure that supervisions are at least two calendar days long.

Decision variables

$$y_{rkt} = \begin{cases} 1, & \text{if route } r \text{ is chosen in Month } k \text{ on travel day } t \\ 0, & \text{otherwise} \end{cases} \qquad r \in R, k \in K, t \in T$$

In the case of Bandundu, with a problem horizon of six months and twelve travel days per month, the solution will specify a set of twelve ordered routes for each month for a total of 72 active routes over the problem horizon.

We introduce the following variables to ensure that each hospital receives exactly one medication delivery during the same travel day every month.

$$q_{it} = \begin{cases} 1, & \text{if hospital } i \text{ receives medication on day } t \text{ every month} \\ 0, & \text{otherwise} \end{cases} \qquad i \in V, t \in T$$

The CVRP-SyncPD model

The model for the CVRP-SyncPD is as follows:

$$\min \sum_{r \in R} \sum_{k \in K} \sum_{t \in T} c_r y_{rkt} \tag{3.1}$$

subject to

$$\sum_{r \in R} \sum_{t \in T} a_{ir} y_{rkt} = 1 \qquad \forall i \in V, \forall k \in K \tag{3.2}$$

$$\sum_{r \in R} \sum_{t \in T} \sum_{k \in K} d_{ir} y_{rkt} \geqslant 1 \qquad \forall i \in V \tag{3.3}$$

$$\sum_{r \in R} d_{ir} y_{rkt} - \sum_{\bar{r} \in R} p_{i\bar{r}} y_{\bar{r}k(t+1)} = 0 \qquad \forall i \in V, \forall k \in K, \forall t \in \{1, 2, ..., |T| - 1\} \tag{3.4}$$

$$\sum_{r\in R} p_{0r} y_{rk1} = 1 \qquad \forall k \in K \qquad (3.5)$$

$$\sum_{r\in R} d_{0r} y_{rk(|T|-1)} = 1 \qquad \forall k \in K \qquad (3.6)$$

$$\sum_{r\in R} y_{rkt} = 1 \qquad \forall k \in K, \forall t \in T \qquad (3.7)$$

$$\sum_{r\in R}\sum_{k\in K} a_{ir} y_{rkt} = |K| q_{it} \qquad \forall i \in V, \forall t \in T \qquad (3.8)$$

$$\sum_{t\in T} q_{it} = 1 \qquad \forall i \in V \qquad (3.9)$$

$$y_{rkt} \in \{0,1\} \qquad \forall r \in R, \forall k \in K, \forall t \in T \qquad (3.10)$$

$$q_{it} \in \{0,1\} \qquad \forall i \in V, \forall t \in T \qquad (3.11)$$

Constraints (3.2) ensure that there is exactly one delivery per month, per hospital. Constraints (3.3) ensure that at least one supervisor visit occurs for each hospital over the problem horizon. Constraints (3.4) ensure that the supervisor is picked up at the same hospital where s/he was dropped off on the previous travel day. Supervisors may remain at a hospital or depot by choosing a route that has the pick-up and drop-off assigned to the same node index (ex. 0PD indicates that the supervisor remains at the depot). Constraints (3.5) ensure that the supervisor is picked up at the depot on the first travel day of every month. Constraints (3.6) ensure that the supervisor is dropped off at the depot on the last travel day of every month. [2] Constraints (3.7) ensure that exactly one route is assigned to a travel day in a given month. We use a null route as a placeholder for travel days in which the vehicle is not actually deployed. Constraints (3.8) ensure that each hospital receives a medication delivery on the same travel day each month (see below for more detail). Constraints (3.9) ensure that the delivery to each hospital is assigned to exactly one travel day.

2. It is feasible for the supervisor to return to the depot before the last travel day and remain there using the null route: (0PD).

Preventing disconnected supervision paths

The path that the supervisor takes over a given month can be obtained by combining the route segments in which the supervisor is moved for each route during the month in the order specified by the travel day index of the decision variables. We call this path the **supervisor path**[3]. Feasible solutions must ensure that the supervisor follows a connected path from and to the depot. The combination of constraints (3.4), (3.5), (3.6), and (3.7) prevent disconnected supervision paths.

1. The supervisor always starts and ends at the depot each month: Constraints (3.5) and (3.6)
2. The supervisor is always picked-up where s/he was dropped-off in the previous travel day: Constraints (3.4)
3. All travel days have a route assigned: Constraints (3.7)

The combination of these constraints prevent disconnected supervisor-paths (including subtours) while allowing the supervisor to return to the depot multiple times and stay at hospital/depot over multiple travel days if necessary.

Ensuring that hospitals receive medications on the same day each month

Constraints (3.8) ensure that each hospital receives their monthly medication delivery on the same travel day of each month. Recall that the coefficient a_{ir} indicates whether a route r delivers to hospital i and that q_{it} allow a constraint row to be active (e.g. with the sum equal to the number of months in the problem horizon $|K|$) or inactive (e.g. with the sum equal to 0), with the constraint being satisfied in either case.

To illustrate, consider a sample solution in Table 3.IV. We have two months, three travel days, and two hospitals. 0 is the depot node, P indicates a pick-up, D indicates a drop-off, and M indicates a medication delivery. So for Month 1 and Day 1 the route is (0P – 25DM – 0): we pick up the supervisor at the depot (0P) and drop him/her off and make a delivery at hospital 25 (25DM), and return to the depot (0). The medication

3. See Figure 5.4 in Chapter 5 for a visualization of a supervisor path.

cluster for Day 1 in both months is a single hospital: 25. The medication cluster for Day 2 in both months is also a single hospital: 31. There is no delivery on Day 3.

	Month 1	Month 2	$\sum_{r\in R}\sum_{k\in K} a_{25,r}y_{rkt}$
Day 1	0P – **25DM** – 0	0P – **25M** – 0D	2
Day 2	0 – 25P – **31DM**– 0	0P – **31M** – 0D	0
Day 3	0 – 31P – 0D	0P – 0D	0

Table 3.IV: A sample solution that satisfies (3.8)

Consider constraints (3.8) for this example. The third column of Table 3.IV illustrates that the solution satisfies constraints (3.8) as the sums are either 0 or $|K| = 2$. Now consider a slightly different solution that satisfies the medication demand each month, but not in the same order each month.

	Month 1	Month 2	$\sum_{r\in R}\sum_{k\in K} a_{25,r}y_{rkt}$
Day 1	0P – **25DM** – 0	0P – **31M**– 0D	1
Day 2	0 – 25P – **31DM** –0	0P – **25M** – 0D	1
Day 3	0 – 31P – 0D	0P – 0D	0

Table 3.V: A sample solution that does not satisfy (3.8)

The last column in Table 3.V illustrates that while the solution satisfies the monthly medication delivery and supervisions constraints (3.2) and (3.3) respectively, it does not satisfy constraints (3.8). In effect, it has delivery to hospital 25 on Day 1 in Month 1 and on Day 2 in Month 2 which violates the problem requirements.

Complexity

The CVRP is well-known to be NP-Hard. Because our problem is at least as difficult as a CVRP, it follows that it is also NP-Hard. It is clear that the complexity for the Set Partition (SP) formulation is dependent primarily on the number of variables because, in contrast to the number of constraints, the number of variables increases as the number of hospital nodes increases. This is due to the exponential number of possible routes we must consider with respect to the number of hospital nodes. Moreover, the novel route attributes (notably the indication of pick-up and drop-off in addition to medication delivery), trip days and months of the CVRP-SyncPD create a significantly larger number of combinations to evaluate compared to a standard CVRP with the same number of client nodes. Consequently, the size of the problem does not allow us to solve the problem exactly. We must consider either a heuristic method that would potentially give sub-optimal results or an exact method that would not require processing all of the routes explicitly.

3.3 Conclusion

In this chapter we described the CVRP-SyncPD as a combined delivery and pick-up/drop-off problem and presented a mathematical model to handle the unique aspects of the combined problem. In the next chapter we present the two-phase heuristic that we used to solve the problem.

CHAPTER 4

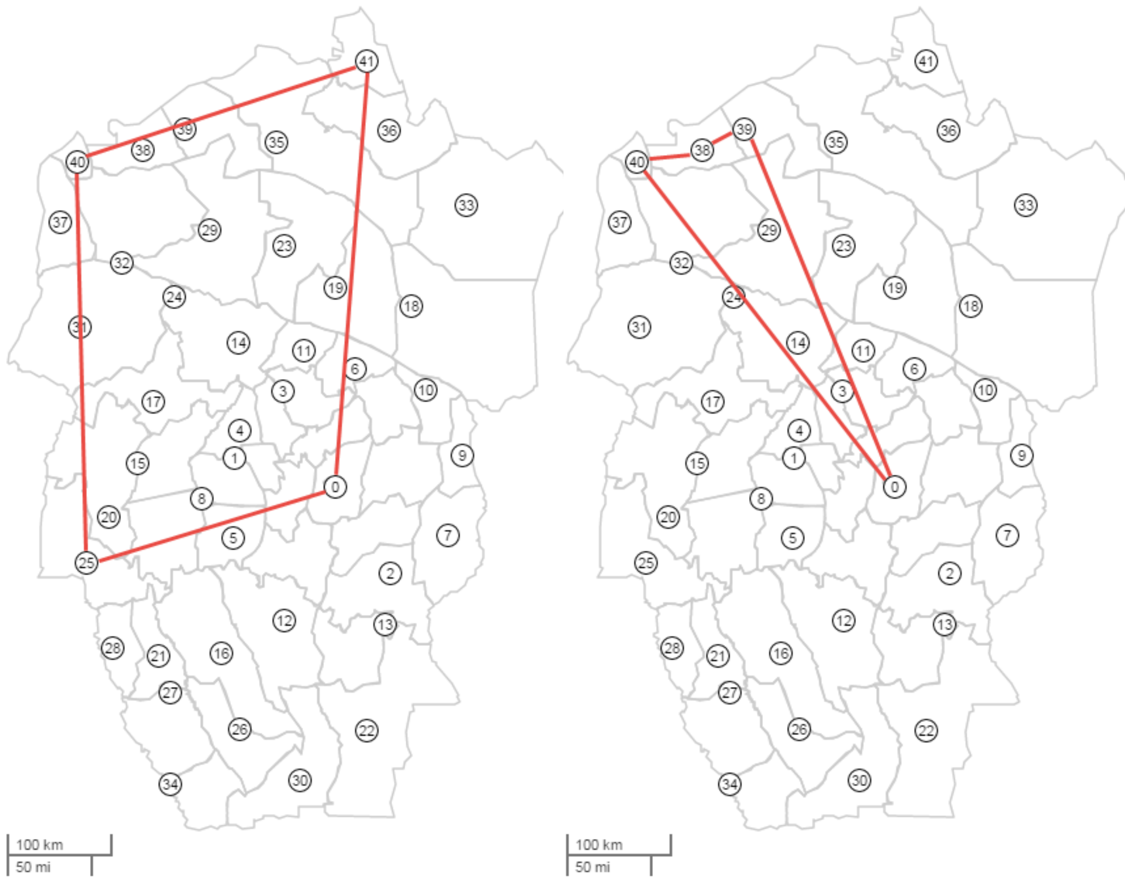
IMPLEMENTATION

In this chapter we present a solution method inspired from the *cluster-first, route-second* approach [23]. Recall that the CVRP-SyncPD must synchronize two tasks (medication delivery and supervisor pick-up/drop-off) to a single route over two frequencies (e.g., once a month for the medication delivery and once every six months for the supervision visit). We constructed a heuristic based on a two-phase cluster-first, route-second approach. **Phase I** is primarily about selecting sets of *clusters* based on set partitions that are used to construct feasible *routes* in **Phase II**. In Phase I we solve a relaxed instance of the CVRP-SyncPD with a subset of medication-only routes using CPLEX. Each solution in CPLEX's solution pool provides a near-optimal routing solution for the relaxed problem. From each solution, we derive a set partition of medication-only clusters. In essence, Phase I solves the medication delivery routing problem with the hypothesis that solutions to the relaxed problem might be a good basis for constructing routes for the CVRP-SyncPD. In Phase II we use the medication clusters from Phase I to construct routes with supervisor pick-ups and drop-offs in order to produce a valid solution to the original problem. We explain Phase I and Phase II in detail in sections 4.1 and 4.2, and provide a description of the software in 4.3.

4.1 Phase I: Select optimal set partitions of medication clusters

Step 1: Construct a subset of routes for the relaxed problem

First, in order to reduce the problem size for our relaxed problem, we construct a limited number of routes based on a subset of medication clusters that contain hospitals that are close to one another. We refer to these as **medication clusters** and their corresponding routes as **medication routes**. This selection criterion allows us to exclude a large number of routes that are very unlikely to be part of an optimal solution of Phase I due to their relatively high route length. For example, the route shown in Figure 4.1a is very expensive because it must connect hospitals that are far apart, while the route shown in Figure 4.1b has relatively low cost because its hospitals are are close together.



(a) Route with hospitals that are far apart (b) Route with hospitals that are close together

Figure 4.1: Comparison of two feasible routes

In order to construct this subset of medication routes, we need to define a metric and a threshold that would determine whether two hospitals are close to each other. There are several measures of proximity that could be used such as: selecting hospitals within a fixed minimum distance from another hospital, selecting the k-nearest hospitals, or partitioning hospitals based on a plane-partitioning method. The existing health zone district map is an example of a plane-partitioning method and allows us to define hospital proximity by whether the border of a health zone geometry (corresponding to a particular hospital) was adjacent to the border of another health zone geometry (corresponding to a different hospital). However, we chose to use a slightly different plane-partitioning method based on a Voronoi diagram that did not require health zones geometries. A Voronoi diagram is a plane partitioning of sections based on distance to a pre-defined set of sites in a specific subset of the plane. For each site there is a corresponding region consisting of all points closer to that site than to any other site [4].

In the context of our problem, a Voronoi diagram partitions the province of Bandundu into cells using hospital locations as sites. In contrast to health zone geometries, the Voronoi cells are constructed solely on the basis of relative distance between sites rather than somewhat arbitrarily-defined political boundaries. As such, we thought they might provide a better measure of hospital adjacency in addition to providing a method for problem sets that did not have the health zone district shapes available.

We used an implementation of the *Sweepline Algorithm for Voronoi Diagrams* by Fortune [15] to generate a Voronoi diagram from the hospital coordinates and defined a hospital as adjacent to another hospital if at least one of the boundaries of their respective cells was adjacent. Figure 4.2 depicts the Voronoi diagram for the hospitals in our problem.

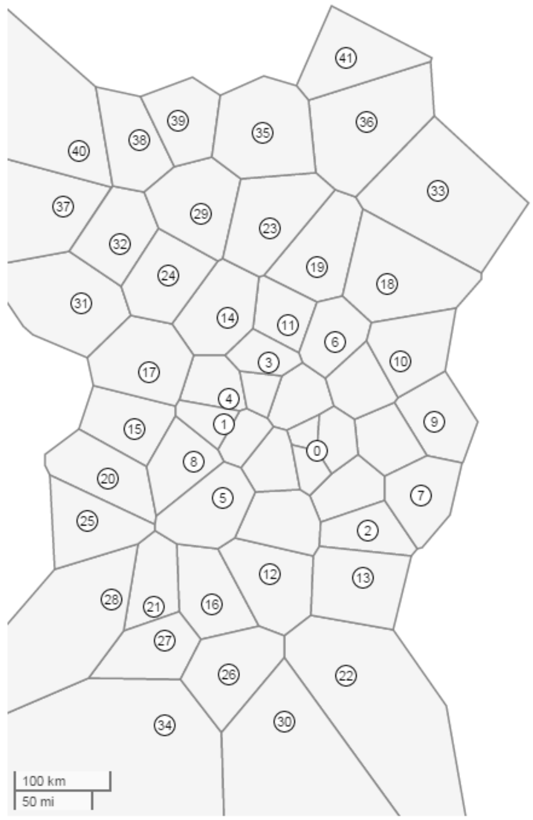


Figure 4.2: Voronoi diagram partition used in our implementation

Using this method, we construct an adjacency matrix that contains the list of adjacent hospitals for each hospital. Then, we construct a set of medication routes by considering all the possible combinations of up to four hospitals, while only including routes from clusters of hospitals that are adjacent to at least one other hospital in the combination. Figure 4.3 shows an example of which hospitals are adjacent to hospital 35 using the Voronoi diagram partition method.

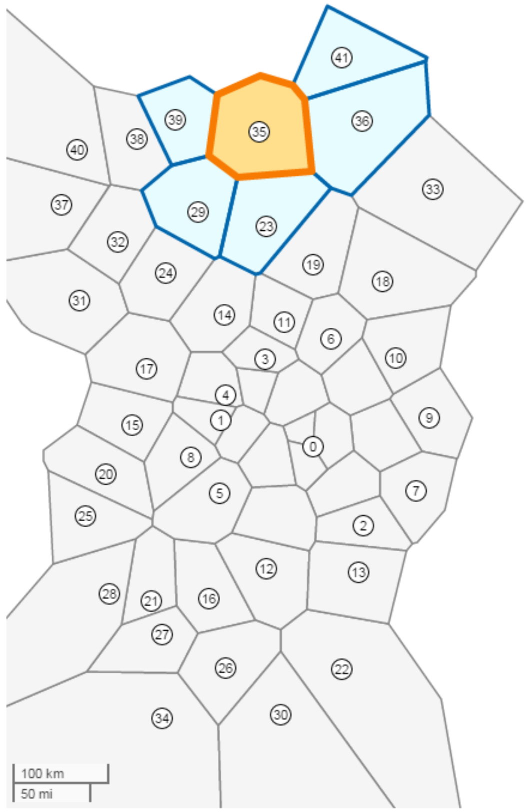


Figure 4.3: An example of how we use the Voronoi diagram to determine hospital adjacency. Voronoi cells for hospitals adjacent to hospital 35 are highlighted in blue.

We construct medication routes by adding the depot node to each adjacent medication cluster, and solving its TSP with Woodbridge's solver [37] to determine the route's visitation sequence, vehicle type, and cost c_r.

Step 2: Model the relaxed problem and generate a solution pool

Because we are not considering travel days, months, or pick-up/drop-off attributes, we can simplify the formulation significantly. The relaxed CVRP-SyncPD formulation solved in Phase I is as follows. Let V be the set of hospitals, $\hat{R}$ be the subset of medication routes, and a_{ir} be coefficients defined as follows:

$$a_{ir} = \begin{cases} 1, & \text{if } r \in \hat{R} \text{ delivers medication to hospital } i \\ 0, & \text{otherwise} \end{cases} \qquad i \in V, r \in \hat{R}$$

Let y_r be the decision variables where:

$$y_r = \begin{cases} 1, & \text{if route } r \text{ is chosen} \\ 0, & \text{otherwise} \end{cases} \qquad r \in \hat{R}$$

Model:

$$\min \sum_{r \in \hat{R}} c_r y_r \tag{4.1}$$

Subject to:

$$\sum_{r \in \hat{R}} a_{ir} y_r = 1 \qquad \forall i \in V \tag{4.2}$$

$$y_r \in \{0, 1\} \qquad \forall r \in \hat{R} \tag{4.3}$$

Constraints (4.2) ensure that there is exactly one medication delivery per hospital.

We solve the problem with CPLEX and generate a pool of near-optimal solutions for Phase I by calling CPLEX's populate solution pool function. The solution pool contains the optimal solution and a specified number of near-optimal solutions. Each solution in the solution pool corresponds to a different set-partition for the relaxed problem. By not limiting ourselves to the single optimal solution in Phase I (medication delivery only), we are able to explore more of the solution space in Phase II (medication delivery and supervision routing) and therefore have the potential to find a better overall solution.

Step 3: Convert routes into clusters for each solution in the solution pool

For each solution in the solution pool, the hospitals found in each route become our medication clusters. For example, the sample solution shown in Figure 4.4 contains 6 routes from which we generate the following set partition of medication clusters {19, 18, 10, 9}, {7, 13, 2}, {12, 16, 5}, {8, 20, 15, 17}, {1, 4}, and {3, 14, 11, 6}.

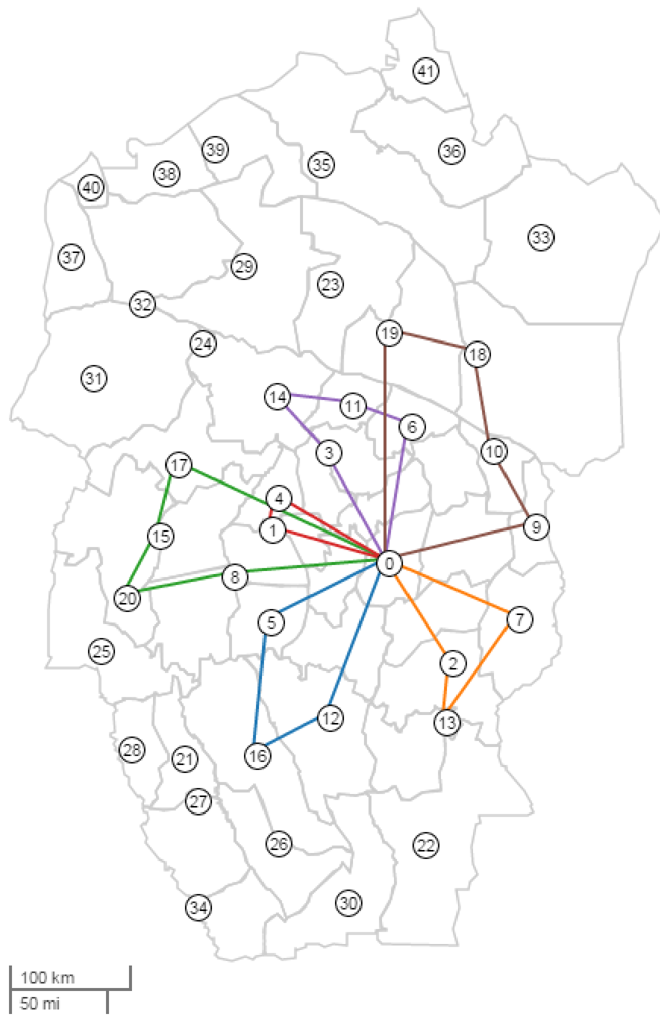


Figure 4.4: Visualization of the optimal routing solution for a subset of 20 hospitals: medication deliveries only.

4.2 Phase II: Construct synchronized medication-supervision routes

In Phase II we use a heuristic to construct synchronized medication-supervision routes (derived from the medication cluster sets we constructed in Phase I) and form a set of complete solutions to the CVRP-SyncPD. This phase contains four steps.

Step 1: Order hospitals based on radian sweeps

Each hospital is assigned a unique radial index starting with 1 based on radial sweeps of its position relative to the depot. The clockwise (forward) ordering is based on a radial sweep about the depot starting at radian 0 as shown by the dotted green line in Figure 4.5. The counter-clockwise order is the inverse of the clockwise order. As such, we have two radial orders under consideration. It is important to note that although we start with an index of 1, this does not imply that the hospital corresponding to this index is the first node in a sequence per se, but rather that it comes between hospital with radial index 2 and hospital with radial index 41, as per the ternary relation in a cyclical order. This allows us to evaluate different initial starting points without re-computing the order each time.

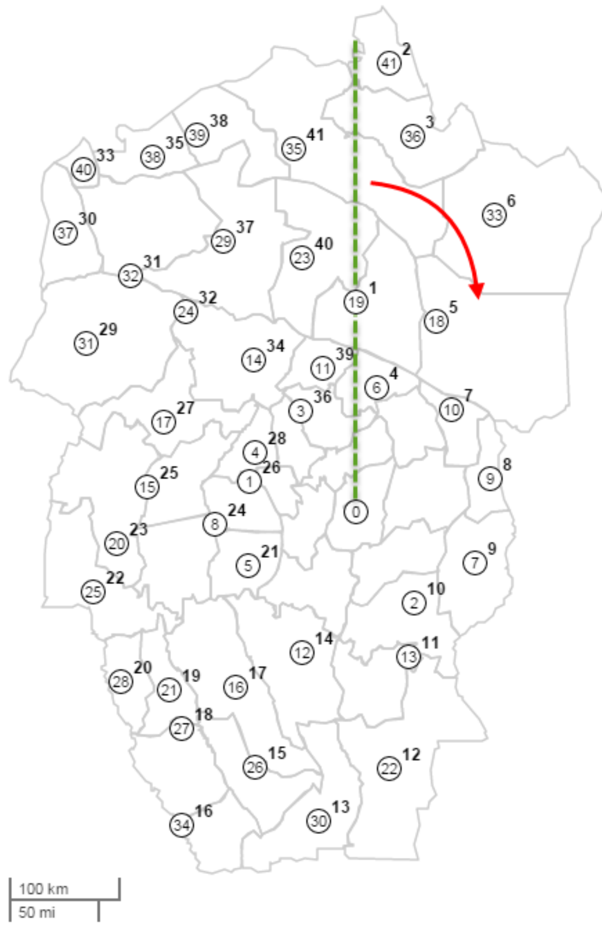


Figure 4.5: The clockwise radial ordering of the 41 hospitals in the problem. The radial order index for each hospital is indicated by the integer just outside the hospital circle.

Step 2: Order medication clusters and hospitals within clusters

For a given set partition of medication clusters, we consider the hospital with the smallest radial index in each cluster and order the cluster using that index. [1] We illustrate a cluster ordering in Figure 4.6 using the example solution for a subset of 20 hospitals. As with the hospital indexing, it is important to note that Cluster 1 is not necessarily the first cluster, it is simply between Cluster 2 and Cluster 6. For each cluster, we add a relative index (shown in red) to each hospital according to the order determined by

1. For clusters with hospitals that straddle 0 radians, we essentially re-index with a different radial initial point than 0 to maintain the ternary relationship.

solving the TSP with Woodbridge's heuristic [37]. For example, the nodes in Cluster 1 {19, 18, 10, 9} are indexed {1, 2, 3, 4} respectively. This often corresponds to the radial indexing (e.g., Cluster 1), but not always (e.g., Cluster 2). In our heuristic this additional indexing specifies the month in which the hospital will be supervised. For example, in Month 1, the supervisor will visit all hospitals with index 1 and in Month 2 the supervisor will visit all hospitals with index 2 and so on. The hospitals that the supervisor visits in a month are sometimes far away from each other, but this simple method allows us to solve the problem quickly and guarantees a solution. We refer to each hospital in its cluster as the k^{th} hospital in that cluster. The subset of hospitals with index k will receive a supervision in the k^{th} month.

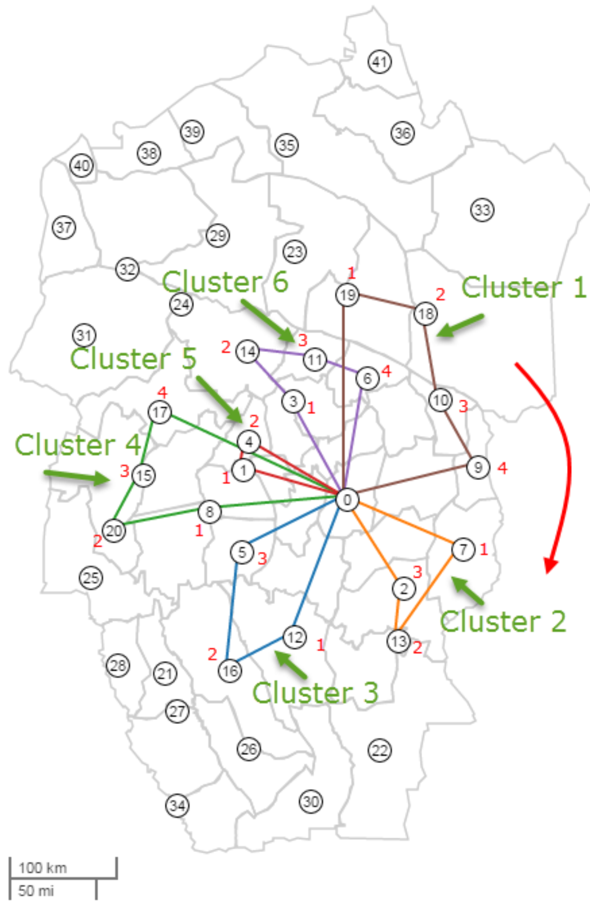


Figure 4.6: Visualization of a clockwise radial ordering of clusters corresponding to an optimal set partitioning of 20 hospitals closest to the depot. The route corresponding to each cluster is also displayed for clarity.

Step 3: Construct synchronized routes and form feasible solutions

The choice of initial medication cluster and cyclical direction (clockwise/counter-clockwise) can affect the total routing cost (in addition to the choice of set partition under consideration from Phase I) so we consider all combinations of these parameters in order to find the cheapest option. For each combination of 1) set partition, 2) initial cluster, 3) and cyclical direction we construct a sequence of synchronized routes for each month to form a feasible solution to the original problem. In essence, each combination defines how we fix several dimensions of the decision space: which hospitals will receive medication for each travel day (for all months) and which hospitals will get supervised in each month (and in what order); what remains is to construct feasible routes that ensure that the supervisor visits the hospitals in the correct order. We accomplish this using a very simple policy that finds the cheapest way to move the supervisor from the depot to the k^{th} node in the initial cluster, then on to the k^{th} node in each of the remaining medication clusters, and finally back to the depot. For each month $k \in K = \{1, 2, ..., 6\}$ we construct a sequence of synchronized routes as follows:

Month 1: On the first travel day, the supervisor is transported from the depot to the first hospital in Cluster 1 and medication is delivered to all hospitals in Cluster 1. On the next travel day, the supervisor is transported from the first hospital in Cluster 1 to the first hospital in Cluster 2 and medications are delivered to each hospital in Cluster 2 (but not necessarily in that order as the route sequence is determined by a TSP). We follow this pattern until the supervisor has visited the first hospital in each cluster and all hospitals have received their monthly medication delivery. After all the clusters have been visited, the supervisor is brought back to the depot.

Month 2: On the first travel day, the supervisor is transported from the depot to the second hospital in Cluster 1 and medication is delivered to all hospitals in Cluster 1. On the next travel day, the aircraft transports the supervisor from the second hospital in Cluster 1 to the second hospital in Cluster 2 and medication is delivered to each hospital in Cluster 2 (but not necessarily in that order as the route sequence is determined by a TSP) and so on until the supervisor has visited the second hospital in each cluster and all hospitals have received their monthly medication delivery. After all the clusters have been visited, the supervisor is brought back to the depot.

Month 3-4: We follow the same pattern as in Months 1 and 2.

Month 5-6: We follow the same pattern as the previous months, but because the cluster size has a maximum of four nodes, we no longer need to transport the supervisor and s/he remains at the depot in all routes.

The number of hospitals in each medication cluster is not always the same. This discrepancy, in combination with the month being considered, affects how we move the supervisor from cluster to cluster. For example, in the set partition shown in Figure 4.6

Cluster 1 contains four hospitals, while Cluster 2 contains three hospitals. Therefore, in Month 4 on travel day 2 we pick up the supervisor from the fourth hospital in Cluster 1, deliver medications to hospitals in Cluster 2 and bring the supervisor back to the depot. It follows that, for months greater than the maximum number of nodes in all medication clusters, supervision is no longer required and the supervisor simply remains at the depot for the entire month.

For some routes it is possible that bringing the supervisor directly from one cluster to another is more expensive than a return flight back to the depot. In those cases, we insert a special route that brings the supervisor back to the depot pending the next travel day. We do not explicitly handle these cases in our MIP formulation because it would add unnecessary complexity to the model; these can be handled post-optimization with minor tweaks to the routes.

In this way, for each month $k \in K$ we generate a sequence of routes (and the associated route costs) in which the supervisor visits the k^{th} hospital in each cluster. Travel days for each route are scheduled in the calendar with an interval of at least two days between each travel day to allow for a complete supervision before the supervisor is moved to the next hospital. The combined set of route sequences for each month constitutes a complete feasible solution to the original problem.

Step 4: Select the cheapest solution

The final step in Phase II is to choose the cheapest solution among those derived using all the combinations following the different set partitions, the different selections of initial cluster, and the different cyclical directions that were constructed in step 3.

4.3 Software infrastructure

Before moving on to a discussion of the results, we take this opportunity to briefly outline the software infrastructure used in our implementation with references to third-party algorithms and software libraries.

We installed the PostgreSQL database software (version 9.3) [2] on a Linux Ubuntu server virtual machine. The PostgreSQL database acts as a datastore for our input data, intermediate data, and solution data. The PostGIS extension [1] to PostgreSQL was installed to give us geospatial functionality that we use extensively in our implementation. Two additional database libraries where installed: a TSP heuristic by Woodbridge [37] and a Voronoi diagram constructor by Fortune [16]. Woodbridge's TSP heuristic was chosen over an exact solver because it was the only known TSP solver to work well in PostGIS. CPLEX was used to solve the relaxed problem in Phase I. We wrote the main algorithm in Python 2.7. We chose Python for the main algorithm over more efficient languages such as Java or C++ because it interfaces easily with PostGIS and CPLEX, and the code is more readily re-usable in the GIS community. The primary role of the main algorithm was to glue together the input and output of various components: user-defined input parameters, CPLEX, and PostGIS.

For visualizations of the solution, we configured a webserver to interface with the database. We wrote a web application that employs leaflet.js [3], D3.js [8] and vivo [14] to obtain results from the database and display them in the browser for further investigation.

4.4 Conclusion

In this chapter we provided a detailed description of the two-phase cluster-first, route-second heuristic approach and the software infrastructure that was used to solve the problem and visualize the results. In the next chapter, we review the results of the implementation.

CHAPTER 5

RESULTS

In this chapter we discuss the results obtained from executing the algorithm in Chapter 4 for Bandundu. In Section 5.2 we present the synchronized routing solution that we found for Bandundu using our heuristic method. In order to help understand the benefits of this solution with respect to existing routing strategies, we describe four alternative routing strategies in Section 5.3 and compare these strategies with respect to the number of flights and the transportation cost in Section 5.4.

5.1 Time to compute a complete solution

On a Windows 7.0 64bit laptop machine with 24Gb of memory and an Intel i7 CPU at 2.4GHz, Phase I took approximately 31 seconds (the route construction for 41 hospitals takes about 29 seconds and CPLEX takes about 1 second to solve the relaxed MIP and produce a solution pool). Phase II takes an additional 30 seconds to evaluate the different combinations and select the cheapest solution.

5.2 Synchronized routing solution for the province of Bandundu

The heuristic generated 2,612 feasible medication-only routes in Phase I and generated five near-optimal solutions in the solution pool. In Phase II, it evaluated routing solutions for combinations over the five set partitions (one for each solution in the solution pool from Phase I), eleven start cluster positions (one for each cluster in a set partition), and two cyclical directions (clockwise/counterclockwise) for a total of 110 combinations. We present the monthly routing solutions for Bandundu in the form of visualizations that were generated by the web-based tool that was developed as part of the implementation. Each visualization illustrates the full synchronized medication delivery and supervision pick-up/drop-off routing for each month as a map and a step-by-step routing table. The table (located to the right of each map) contains detailed information about each route. The descriptions for each abbreviation used in the maps are provided in Table 5.I.

Abbreviation	Description
meds_only	Medication delivery only, no supervision
meds_only_short	Medication delivery only because cluster size is smaller than current month number, no supervision
short_bring_back	Supervisor is currently in the field, but current cluster has less nodes than current month number, so we just bring the supervisor back to the depot
first_normal	First route in month, normal synchronization
first_short	First route in month, no supervisor movement because cluster size is shorter than current month number
piggyback_depot	Pick up supervisor from previous cluster and drop off at some node number in current cluster
piggyback_hospital	Pick up supervisor from depot and drop off in current cluster
return_to_depot	Pick up supervisor from last cluster and drop off at depot

Table 5.I: Route type abbreviations and descriptions

Figure 5.1 shows a sample of how the route details are displayed in the tables. The first element of the route is the travel day (e.g., 5). The second element of a route is the abbreviated route type (e.g. piggyback_hospital). The third element is the aircraft type and the third and fourth elements are total route distance and total route cost respectively.

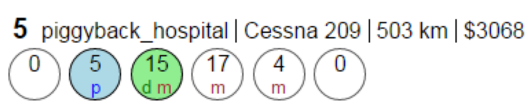
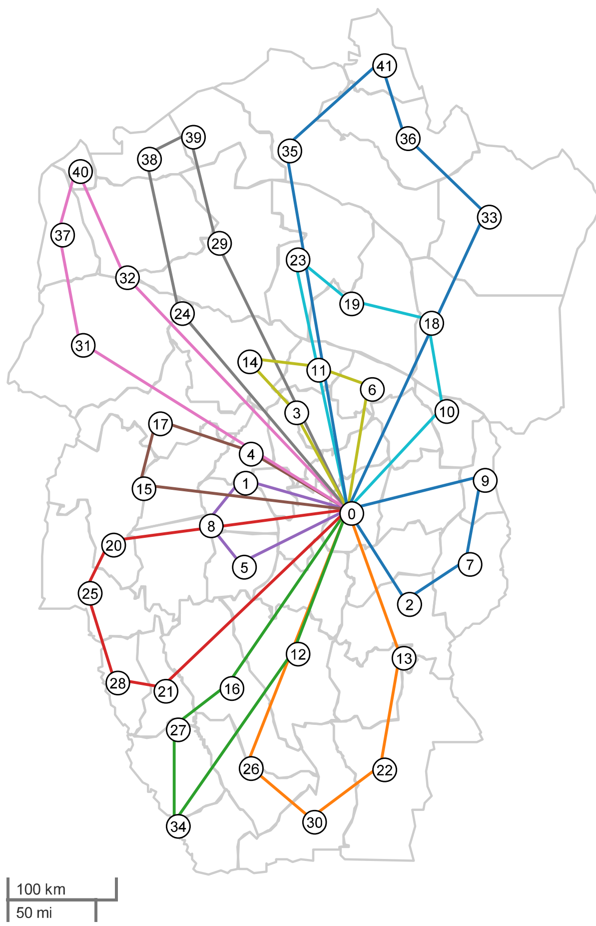


Figure 5.1: Sample route details

The circles correspond to the nodes' numbers on the map. A blue circle with a 'p' signifies a pick-up of the supervisor, a green circle 'd' signifies a drop-off of the supervisor and an 'm' signifies a medication delivery at that node. In the example shown in Figure 5.1, on travel day 5 of the month, a Cessna 209 departs from the depot (node 0), picks up the supervisor at hospital 5, drops the supervisor off at hospital 15 and completes a delivery, then travels to hospital 17 to make a delivery, travels to hospital 4 to make a final delivery and then travels back to the depot.

Figure 5.2 shows the routing that is used for months when we have no supervision (i.e. only months five and six because the supervisor will have visited all the hospitals in the first four months). We include it first as the routes were generated in Phase I and subsequently used to determine the clusters for the combined routes in Months 1, 2, 3, and 4 that follow. The solution indicates that the starting medication cluster was {9, 7, 2} with a clockwise cluster order. The map and table were captured from a web browser displaying the solution via the web-based interface tool.

medication-only-month



1 meds_only | Cessna 209 | 382 km | $2330	0	9 m	7 m	2 m	0
2 meds_only | Cessna 209 | 678 km | $4140	0	13 m	22 m	30 m	26 m	0
3 meds_only | Cessna 209 | 702 km | $4280	0	12 m	34 m	27 m	16 m	0
4 meds_only | Cessna 209 | 661 km | $4030	0	21 m	28 m	25 m	20 m	0
5 meds_only | Cessna 209 | 323 km | $1970	0	5 m	8 m	1 m	0
6 meds_only | Cessna 209 | 466 km | $2840	0	15 m	17 m	4 m	0
7 meds_only | Cessna 209 | 892 km | $5440	0	31 m	37 m	40 m	32 m	0
8 meds_only | Cessna 209 | 837 km | $5110	0	24 m	38 m	39 m	29 m	0
9 meds_only | Cessna 209 | 413 km | $2520	0	3 m	14 m	11 m	6 m	0
10 meds_only | Cessna 209 | 608 km | $3710	0	23 m	19 m	18 m	10 m	0
11 meds_only | Cessna 209 | 963 km | $5870	0	35 m	41 m	36 m	33 m	0

Figure 5.2: Visualization of the medication-only routing solution for Month 5 and 6.

Figure 5.3 shows the routing solution for Month 1. Travel day 11 illustrates one of the benefits of synchronization: adding hospital 23 to the medication-only route in order to pick up the supervisor from the previous cluster results in a very small deviation that does not even increment the total distance by a single kilometre (remains at 963 km).

Month 1

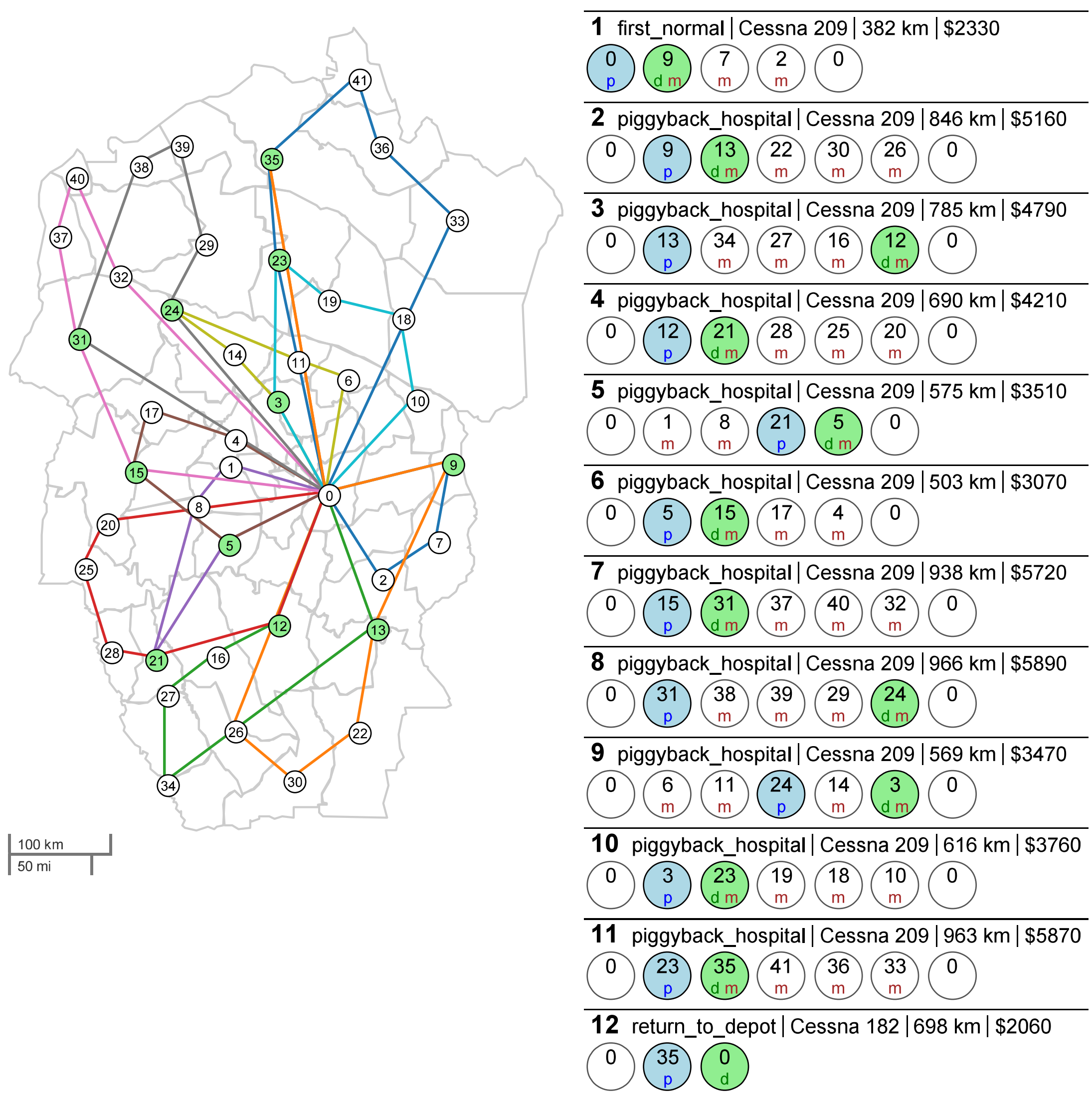


Figure 5.3: Visualization of the full routing solution for Month 1.

The map in Figure 5.4 highlights the path the supervisor takes in Month 1 (other route segments are intentionally hidden). In travel day 3, we show that the supervisor moves from hospital 13 to 12 via hospitals 34, 27, and 16 in order to deliver medications. These extra short stops are actually beneficial as each stop gives the supervisor an opportunity to meet some of the hospital staff during unloading and potentially exchange documents and funds.

Month 1

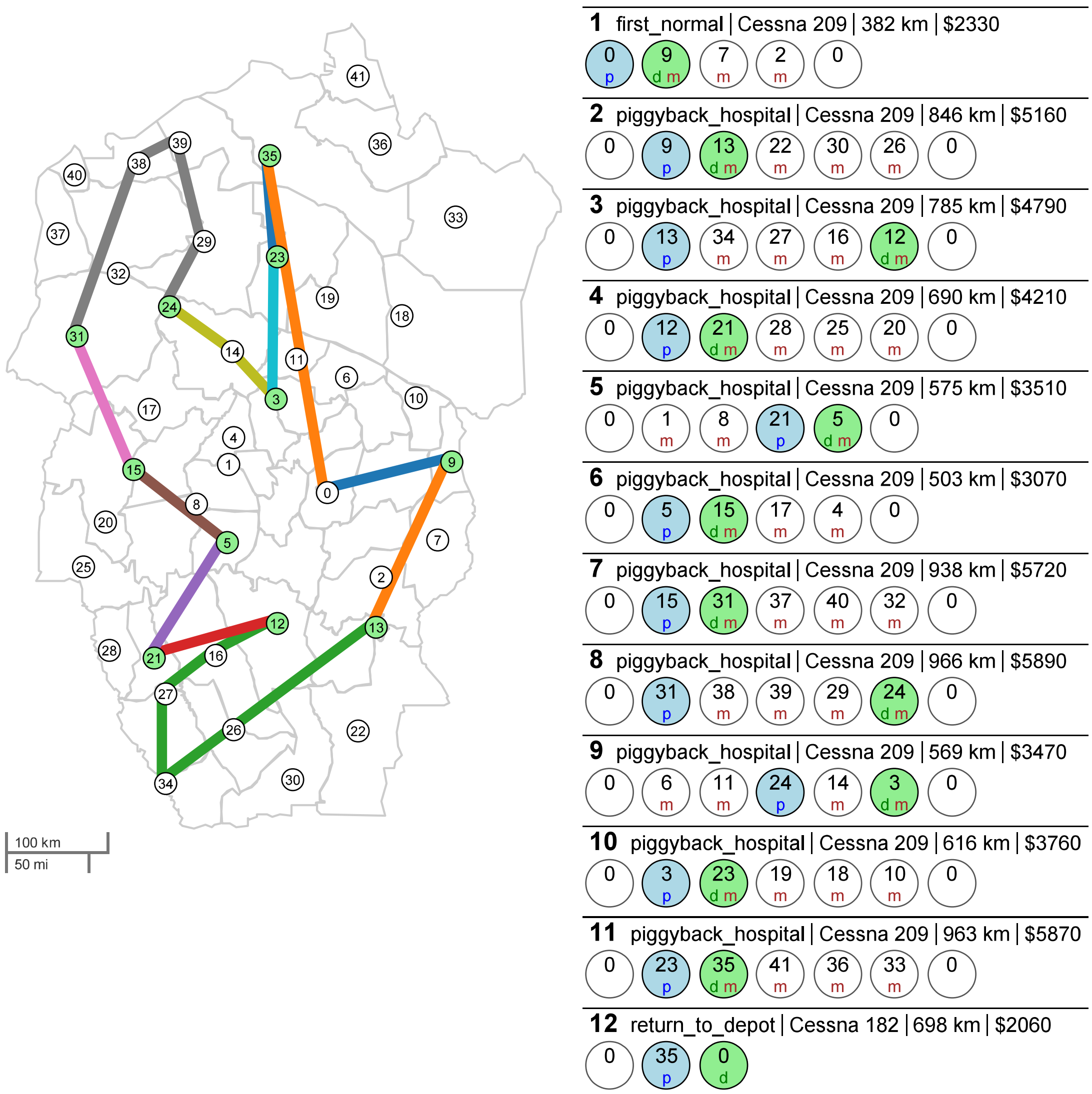


Figure 5.4: Visualization of the supervisor path in Month 1.

Figure 5.5 shows the full routing solution for Month 2. On travel day 7 we see another instance of cost-savings from synchronization: picking up the supervisor from hospital 17 adds just five kilometres to the original medication-only route (892 km versus 897 km).

Month 2

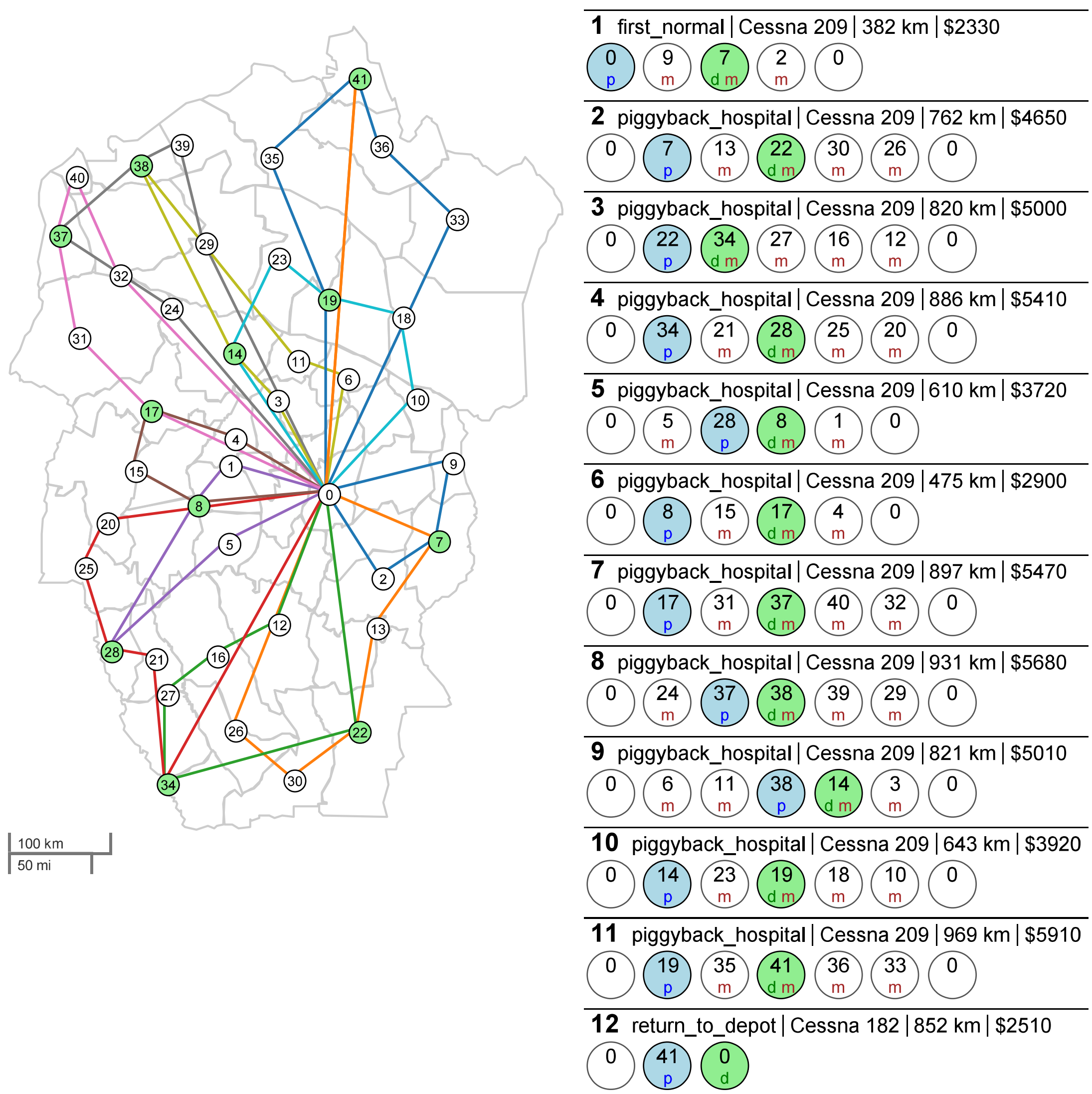


Figure 5.5: Visualization of the full routing solution for Month 2.

Figure 5.6 highlights the supervisor path in Month 2.

Month 2

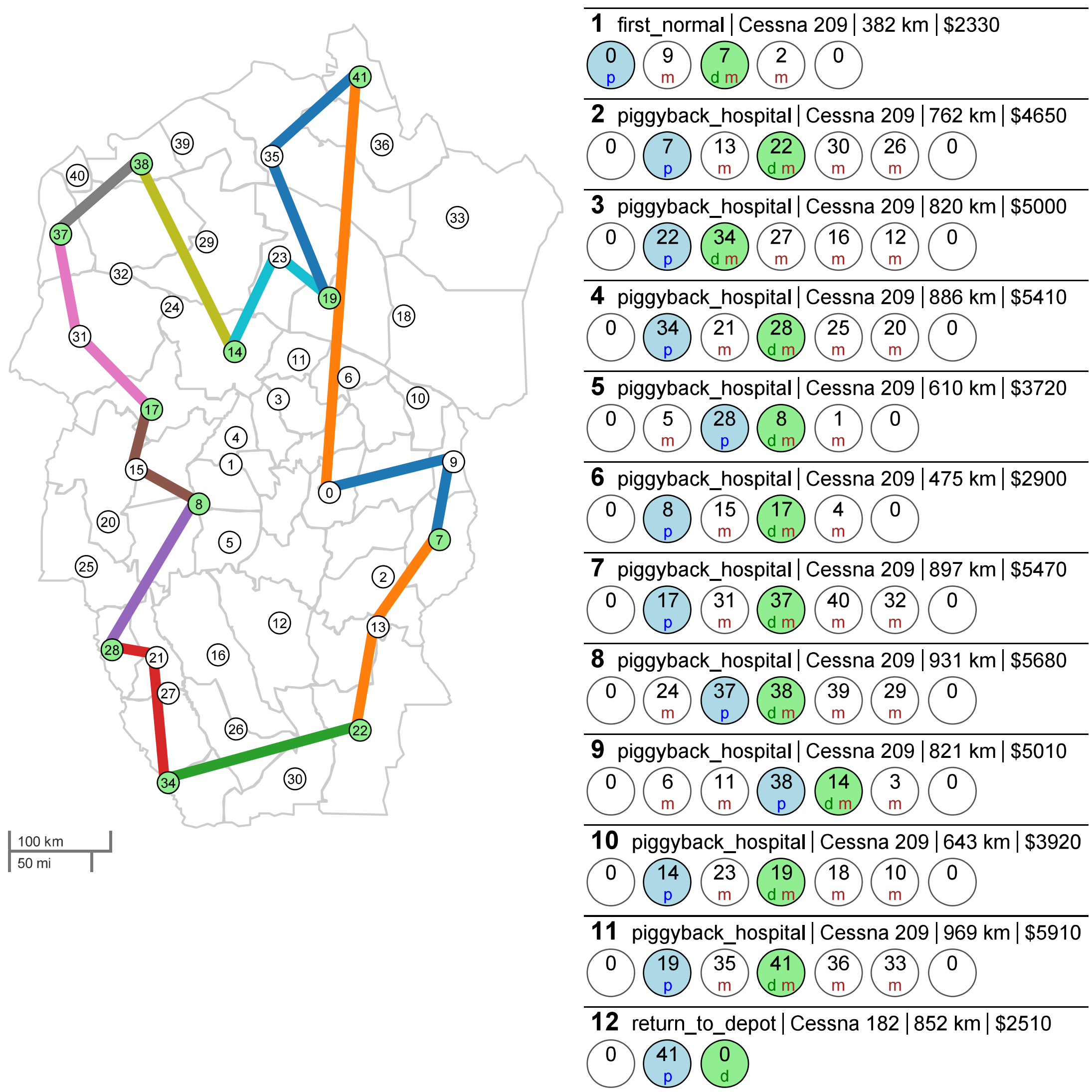


Figure 5.6: Visualization of the supervisor path in Month 2.

Figure 5.7 shows the solution for Month 3. We see that the route sequence in travel day 1 is automatically reversed from previous months in order to minimize the length of the supervisor path: (2, 7, 9) instead of (9, 7, 2). However, the supervisor could choose to maintain the original sequence without affecting the objective value of the solution because the total route distance remains the same.

Month 3

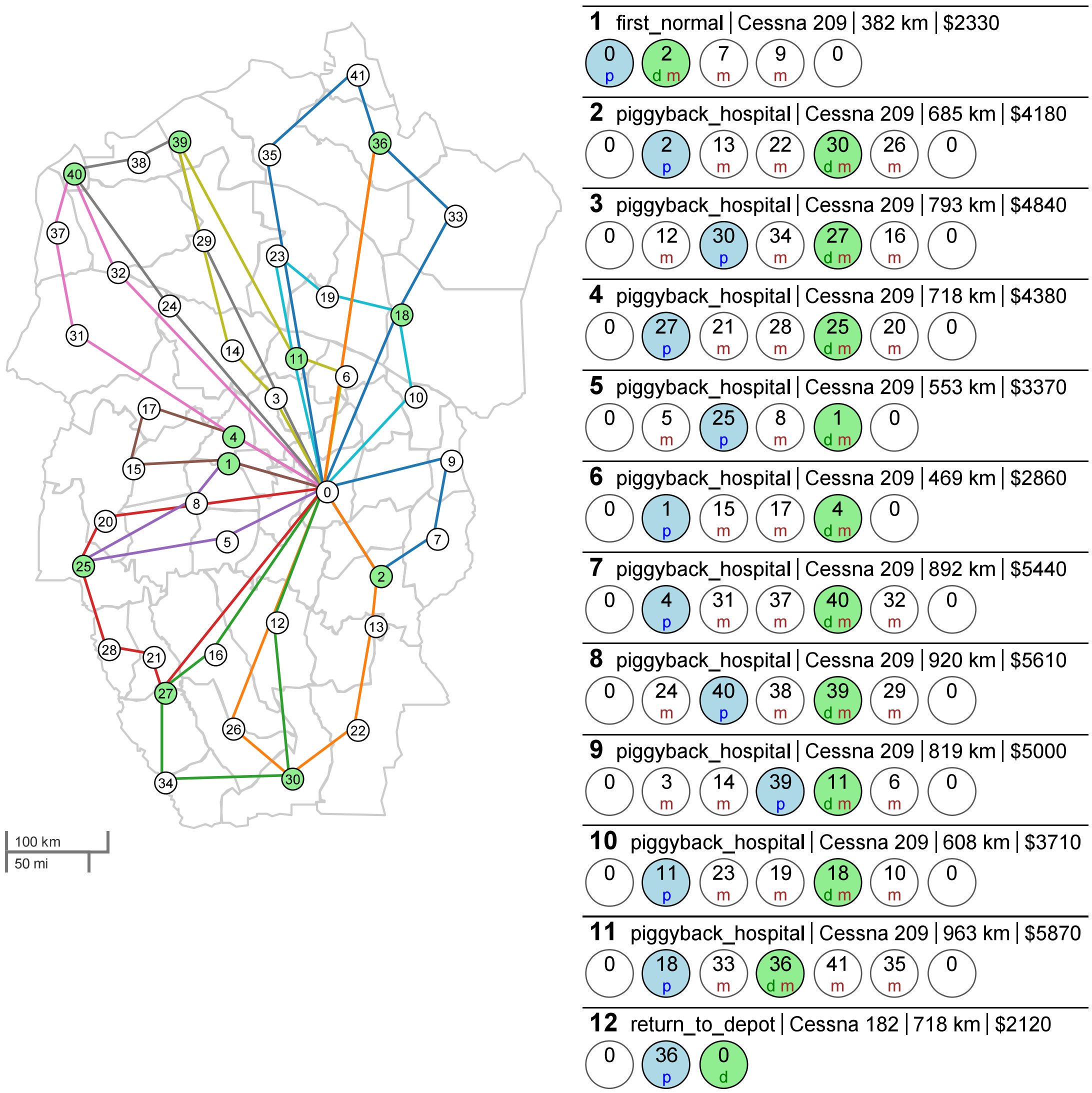


Figure 5.7: Visualization of the full routing solution for Month 3.

Figure 5.8 highlights the supervisor path in Month 3. In travel day 7, we can observe that the supervisor travels quite a distance even though s/he moves from hospital 1 to 4 which are quite close together. This is because we are concerned only with the total distance the plane has to travel and in this case the TSP solution indicated that this was the cheapest path.

Month 3

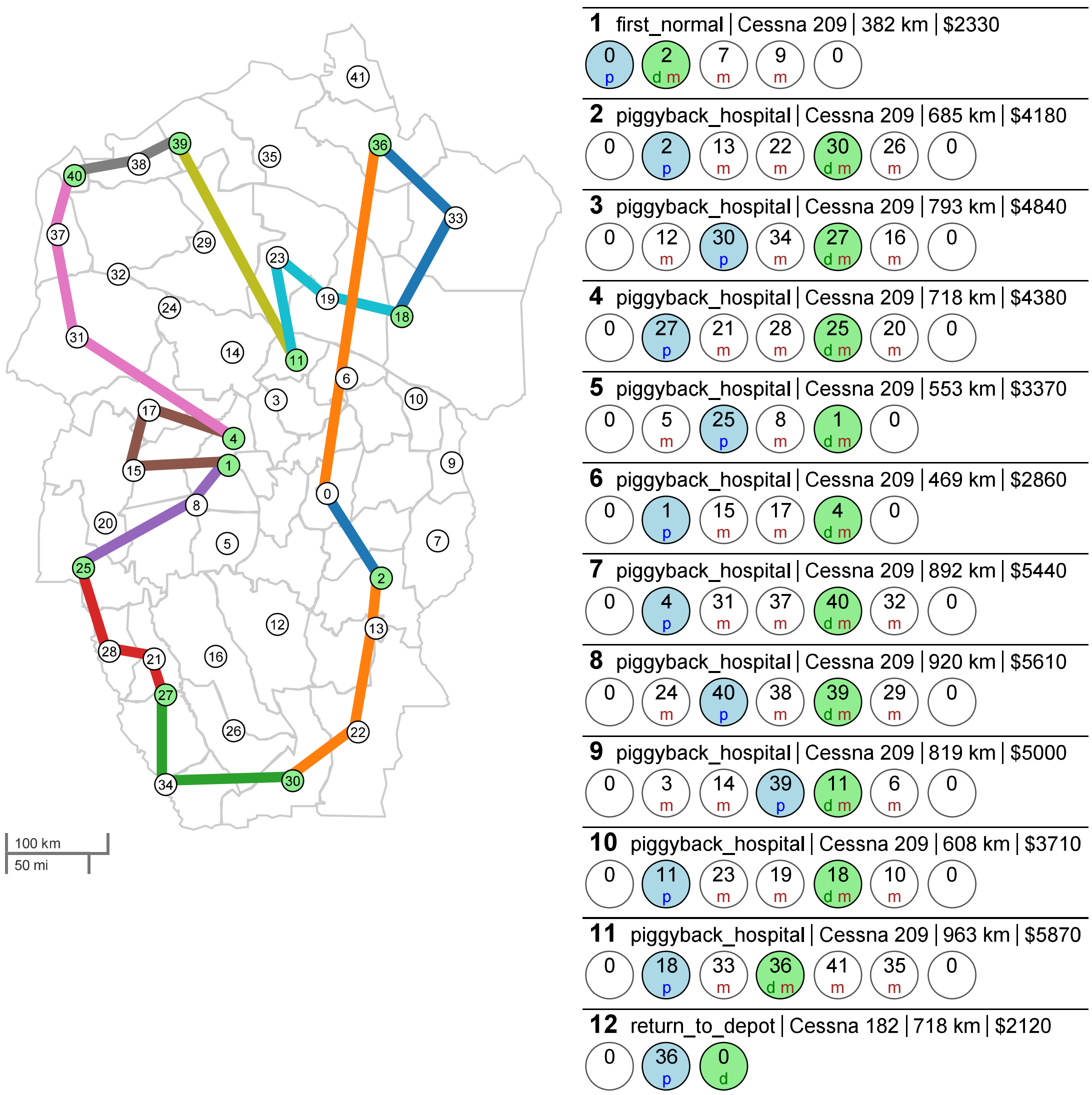


Figure 5.8: Visualization of the supervisor path in Month 3.

Figure 5.9 shows the synchronized routing solution for Month 4. Travel day 3 illustrates another interesting route with respect to cost-savings as the route only increases the total distance by 7 km compared to the original medication-only route (709 km versus 702 km).

Month 4

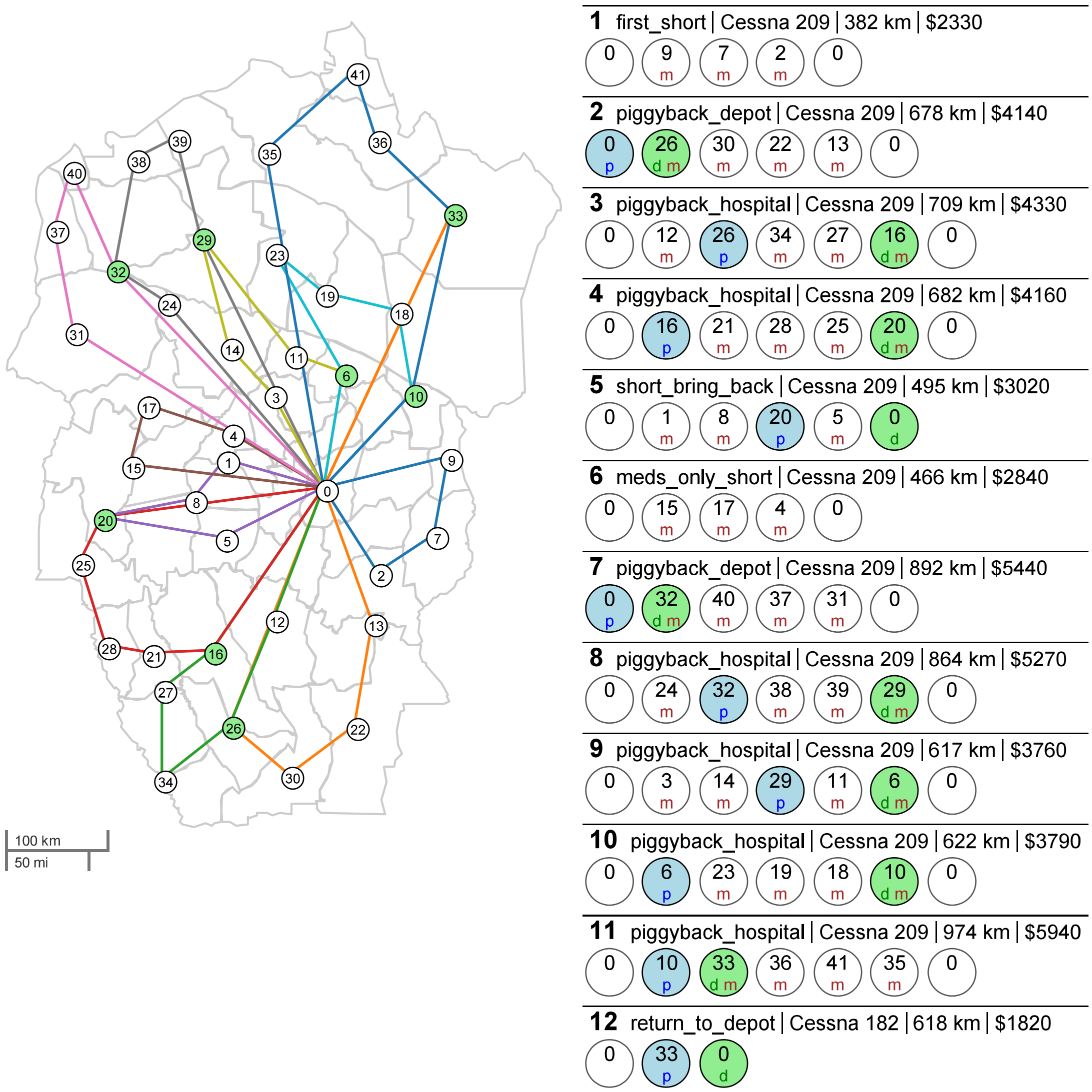


Figure 5.9: Visualization of the full routing solution for Month 4.

The visualization of the supervisor path in Month 4 shown in Figure 5.10 illustrates how the heuristic handles moving the supervisor with mismatched route lengths. For example, on travel day 1 there is no supervision because there are only three delivery nodes in the associated cluster and because we are in the 4th month, we are supervising the 4th nodes of each cluster. On travel day 5, we bring the supervisor back to the depot from hospital 20 because there is no supervision required on travel day 6.

Month 4

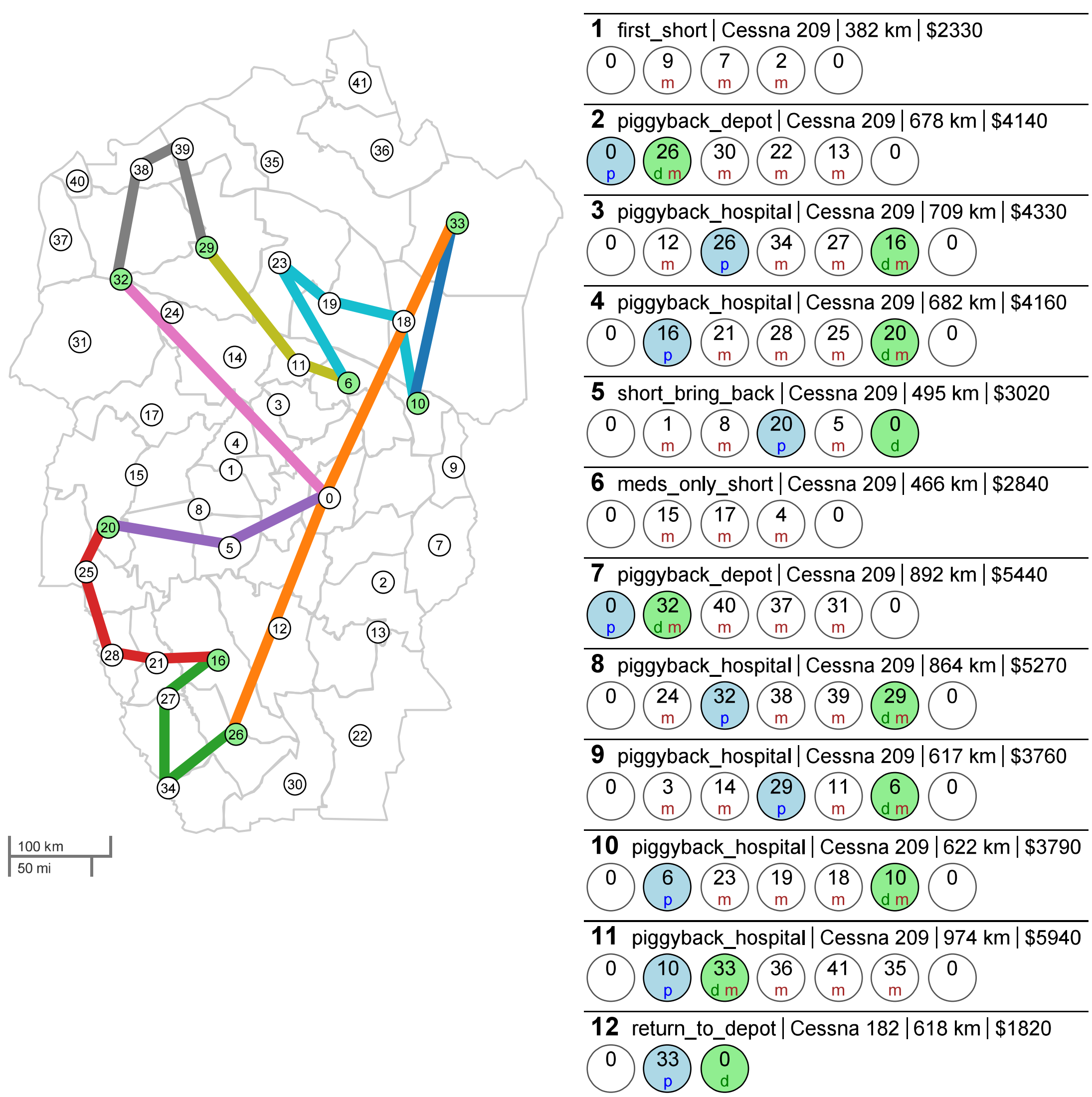


Figure 5.10: Visualization of the supervisor path in Month 4.

The total routing costs of the synchronized solution for each month were computed by adding the individual route costs and are provided in Table 5.II. The average routing cost per month per hospital is $1,152. We use this metric to compare alternative strategies. In the months in which we synchronize supervision with medication delivery (Months 1 through 4) the cost is at most 24.3% more (in Month 2) than the months where the supervisor remains at the depot (Months 5 and 6). The average monthly routing cost ($47,230) is only 11.8% more than the months with medication-only routing.

	Routing cost
Month 1	49,840
Month 2	52,510
Month 3	49,710
Month 4	46,840
Month 5 (medication delivery only)	42,240
Month 6 (medication delivery only)	42,240
Total	283,380
Average per month	47,230
Average per month per hospital	1,152

Table 5.II: Total routing synchronized routing costs for each month for Bandundu

5.3 Alternative strategies

The solution we generated for the CVRP-SyncPD corresponds to one routing strategy available to decision makers. Now that we have generated a solution for the CVRP-SyncPD in the case of Bandundu, we can compare it to alternative strategies that decision makers have at their disposition. In this section we describe the four strategies under consideration roughly in order of complexity starting with the easiest to compute and progressing to the most difficult to compute. We then summarize the salient properties of each in Table 5.III.

The simplest but most expensive strategy would be to find independent solutions to the two sub-problems with no attempt to coordinate the routes. We call this the No coordination strategy **(no-coord)** because no coordination is required between medication delivery and supervisory visits. We send large orders of medications to each hospital every four months and stock outages are handled via **emergency orders**. An emergency order occurs when there is a stock outage of an essential medication and local hospital administration decides that they must procure the medication before the next regular order. Although we do not have hard data on the cost of emergency orders we know that they are significant and especially so for remote hospitals. Supervisors are dropped off and picked up in a small aircraft independently from the medication deliveries. The cost of this strategy is easy to calculate: one return medication delivery trip to each hospital every four months and two return supervision trips every six months.

A slightly cheaper strategy but still simple to compute, is to deliver a large order of medications to each hospital every four months but also send the supervisor with the delivery and schedule a single return flight for the supervisor. We call this the Minimal coordination strategy **(min-coord)**. The cost is less than the no-coord strategy because we eliminate one return flight per hospital. However, it is still sub-optimal from a pharmacy management perspective due to the large interval between orders and the cost of emergency orders.

The simplest strategy with monthly delivery is called the Monthly medication delivery with asynchronous supervision strategy **(radial-async)**. In this strategy, medication delivery is routed to all hospitals every month using the medication-only solution from Phase I of our heuristic. Emergency orders are not typically required due to the monthly order frequency. The supervisor is deployed to each of the hospitals via a medication flight and returns in the same way as in min-coord: an aircraft is scheduled to bring him/her back to the depot after each supervision. This strategy requires solving a Phase I problem using a MIP solver, but it is relatively easy to solve because there are no supervision synchronization requirements.

The final strategy and the most difficult to solve, is called the Synchronized medication delivery and supervision pick-up and drop-off strategy **(radial-sync-K)** where K is the number of months in the problem horizon. We consider two instances of this strategy, radial-sync-4 and radial-sync-6, in order to give additional context to the reader. However, our primary interest is with radial-sync-6 because the original problem requires that a supervisor visits each hospital at least once every six months. With this strategy we synchronize the supervision pick-up and drop-off and medication tasks over the problem horizon. It is the cheapest synchronized option with monthly deliveries but it requires solving a Phase I and II Decision tool as described in our heuristic method.

Strategy	Description	Delivery interval	Decision tool
no-coord	Supervision routing independent from medication delivery	4 months	None
min-coord	Supervisor travels with medication delivery every four months and is picked-up two calendar days afterwards	4 months	None
radial-async	Supervisor travels with medication delivery every month and is picked up two calendar days afterwards	1 month	Phase I
radial-sync-4	Supervision synchronized with medication delivery every month over four months	1 month	Phase I and II
radial-sync-6	Same as radial-sync-4, except over six months	1 month	Phase I and II

Table 5.III: Summary of the routing strategies we considered

5.4 Comparison of the strategies

We compare each of the strategies using two metrics: the average number of flight-s/month and the average cost of medication delivery and supervision.

In Table 5.IV we present the number of flights for each strategy. The number of flights is an important metric because there are administrative costs incurred for each flight regardless of the distance travelled. We shall refer to these costs as **start-up costs**. For example, there are take-off and landing fees that must be paid to the airport authorities. We do not have sufficient data to estimate these costs and they vary depending on the context and consequently we do not include them explicitly in our cost analysis. However, exposing the number of flights for each strategy provides an important basis for comparison.

	Frequency (months)		Number of flights				
	Med. Delivery	Supervision	Med only	Med. + Sup.	Sup. only	Total/ Year	Avg/ month
no-coord	4	6	41	0	82	287	23.9
min-coord	4	4	0	41	41	246	20.5
radial-async	1	4	0	44	41	255	21.3
radial-sync-4	1	4	0	44	4	144	12.0
radial-sync-6	1	6	0	66	4	140	11.7

Table 5.IV: Number of flights breakdown for each strategy

The **no-coord** strategy, delivers medications every four months and supervisors visit hospitals every six months. This results in 41 medication delivery flights every four months and 82 supervision flights to bring the supervisor to and from each hospital every six months. There are no synchronized flights. Multiplying the number of flights by their respective frequencies gives 287 flights per year with an average of 23.9 flights per month. The **min-coord** strategy sends the supervisor out with the medication flights every four months for a total of 41 medication+supervision flights and retrieves the supervisor in a small plane for an additional 41 supervision only flights. That makes 246 flights per year and an average of 20.5 per month. The **radial-async** strategy requires eleven monthly medication deliveries each month for a total of 44 medication flights over four months. The supervisor travels with a medication flight and gets picked up in a small plane once the supervision is completed which adds 41 flights every four months. That makes a total of 255 flights per year and an average of 21.3 per month. The **radial-sync-4** strategy solution for Bandundu has eleven synchronized flights per month for a total of 44 and an additional flight at the end of each month to return the supervisor to the depot. That makes a total of 144 flights per year and an average of twelve per month. Finally, the **radial-sync-6** has synchronized flights every month over four months, and the remaining two months are medication-only flights. That makes a total of 140 flights per year with an average of 11.7 per month. The main conclusion to draw from this table is that the min-coord strategy requires almost twice as many flights/month than the synchronized radial-sync-6 strategy (20.5 versus 11.7). We do not explicitly take the start-up cost of each flight into consideration when evaluating the costs because we do not have enough data. However these costs are not negligible and the number of flights will be considered by decision makers when evaluating strategies.

Table 5.V shows the breakdown of the average transportation cost (per hospital, per month) for both medication delivery and supervision pick-up/drop-off for each strategy. The average cost of medication delivery and supervision for the **no-coord** and **min-coord** strategies was computed by summing the route costs for the required flights based on the total distance and cost per kilometer. For example, for the no-coord strategy we have medication-only flights to each of the 41 hospitals every four months which require the Cessna 209 aircraft at \$6.1/km at a total cost of \$116,808 and two supervisor-only flights to each hospital that require the Cessna 182 aircraft at \$2.95/km every six months at a total cost of $\$56,489 \times 2 = \$112,978$ (See Table V.I in Appendix V for details). Taking the average of these costs over twelve months and for each hospital gives \$1,172 per hospital per month. For the remaining strategies, we used the synchronized solution that our heuristic generated to calculate the costs. From Table 5.II we know that the average monthly cost of the radial-sync-6 is 1,152 per hospital. We can determine the part of the total cost attributed to medication by looking at the cost of a month with no supervision (Month 5 and 6) and dividing by the number of hospitals $42,240 \div 41 =$ \$1030.

Strategy	Average cost of delivery and supervision in \$	Medication delivery portion \$ (%)	Supervision pick-up/drop-off portion \$ (%)	Increase from baseline \$ (%)
no-coord	1,172	712 (61%)	460 (39%)	115 (11%)
min-coord	1,057	712 (67%)	345 (33%)	-
radial-async	1,259	1,030 (82%)	229 (18%)	203 (19%)
radial-sync-4	1,213	1,030 (85%)	183 (15%)	156 (15%)
radial-sync-6	1,152	1,030 (89%)	122 (11%)	95 (9%)

Table 5.V: Comparison of average transportation cost/month/hospital for medication delivery and supervision pick-up/drop-off

There are four main conclusions to draw from this table. First, the synchronized solution (radial-sync-6) with monthly deliveries costs only \$95 (9%) more than the baseline strategy (min-coord) which delivers medication to each hospital only once every four months.

Second, although the radial-sync-6 strategy costs \$95 more on average than the min-coord strategy for transportation costs, we are not factoring in the start-up costs for each flight or the cost of emergency orders in this analysis due to lack of hard data. Start-up costs and emergency orders will increase the total cost of the min-coord strategy because this strategy requires more flights and emergency orders, but the degree of the cost increase is uncertain. Even in the absence of hard data on these additional costs, we can observe that it would only take an increase of \$95 (per hospital per month) for the min-coord strategy to break even with the radial-sync-6 strategy. According to Menno-Santé coordinators, the average start-up costs and emergency orders for the min-coord strategy could easily exceed this amount. As such, we estimate that the total cost of min-coord strategy is not significantly more expensive (if at all) than the radial-sync-6 strategy.

Third, the supervision portion of the radial-sync-6 strategy is only \$122 (11%) of the total transportation cost compared to \$345 (33%) with the min-coord strategy. This suggests that if monthly medication deliveries were deemed necessary, adding supervision pick-up/drop-off to the monthly medication-only routing would also be easy to justify.

Fourth, the difference between the cost of the radial-async and radial-sync-6 $(1,259 - 1,152 = \$107)$ is the portion of savings accumulated during Phase II. In other words, if administrators only used Phase I of our algorithm (or a standard petal-based approach) to produce a solution for the radial-async strategy, they would be foregoing a significant portion of cost-reduction due solely to the synchronization of the supervision and medication delivery activities.

The main conclusion to draw from these results is that by synchronizing the tasks of our combined medication delivery and supervision problem hospitals can benefit from the advantages of monthly medication delivery (fewer stock outages and less expired medications) at virtually no additional cost to hospitals.

5.5 Conclusion

We have presented the solution to the problem for the province of Bandundu. We have illustrated that it is possible to find an approximate solution to the combined delivery and supervision problem in a reasonable time-frame. We have also shown that although the transportation costs of a synchronized solution (radial-sync-6) are slightly more than the baseline strategy (min-coord), when we consider the start-up costs for each flight and the emergency orders required by the min-coord strategy, the cost difference is not significant. Consequently, our heuristic provides a feasible solution to a synchronized strategy which offers a monthly medication delivery with supervision at virtually the same total cost than the current min-coord strategy. This is an exciting finding that could provide justification and the technical means for practitioners to provide monthly medication delivery and offer regular supervision at virtually no additional cost. It also illustrates a novel example of combining heuristics with GIS and interactive optimization techniques by way of commonly available technology and an accessible web-based interface.

In the next and final chapter, we review the contributions of this thesis and outline possible directions for further research.

CHAPTER 6

CONCLUSION

In this chapter we summarize the main contributions of the thesis, and suggest directions for further research.

6.1 Contributions

Humanitarian logistics

This work contributes to the existing Humanitarian Logistics literature by exposing, describing, and modelling a synchronized medication and supervision routing problem that is well-known to practitioners but under-researched by theoreticians. Furthermore, we provided sufficient illustration of the concept for an eventual GIS-enabled web-based decision tool for practitioners in the field. This research specifically contributes to the sub-field of Humanitarian Logistics concerned with logistics problems categorized as post-emergency reconstruction.

CVRP

We contributed to the body of CVRP literature by introducing 1) a formulation for the CVRP-SyncPD, 2) a novel class of synchronization over two activity frequencies 3) an efficient heuristic method for solving the CVRP-SyncPD, and 4) a novel interactive visualization tool for post-optimization exploration and analysis.

Contribution to practitioners

Our results from examining the Bandundu case show that while the min-coord strategy is the cheaper option when considering direct transport costs only that a monthly synchronized strategy is also a viable option. If practitioners want to increase the frequency of medication orders to once per month in order to improve availability, they now have the technical means to find the routing solution. In problem instances in which medication cluster sizes can be reduced to two or three nodes (e.g. by increasing monthly demand), or if the nodes were more densely positioned, we think it is possible that the radial-sync-6 strategy would cost even less than the min-coord strategy.

6.2 Further research

Avenues of research that could extend the work of this thesis fall broadly into five categories: 1) extensions to the variable space, 2) improvements to the model formulation, 3) improvements to the heuristic method, 4) improvements to the algorithm implementation, 5) improvements to the web-based interface, 6) commercial applications, and 7) assessing applicability based on cluster density.

Extensions to the variable space

Currently the model handles a single supervisor and a single depot, and we require that the supervisor returns to the depot at the end of each month. We propose two ideas for extensions to the variable space. First, the model could be extended to include multiple supervisors and depots. Apart from allowing multi-province optimization, this would increase the number of travel days available per month by allowing multiple supervisors to be deployed at once. However, adding depots and supervisors will increase the number of variables substantially. Second, we could consider allowing a continuous horizon in which supervisors are not required to return to the depot each month.

Improvements to the model formulation

The Set Partition model formulation for the CVRP-SyncPD presented in Chapter 3 can possibly be improved by adding additional cuts such as Valid Inequalities to reduce the solution redundancies related to the fact that the month ordering is not relevant. The techniques presented by Baldacci and Christofides [5] could be adapted to reduce the problem size even further.

Improvements to the heuristic method

The heuristic method performance and solution quality could be improved in several ways. One way would be to traverse the radial ordering and cluster index selection combinations implicitly (rather than explicitly) by way of a linear program or by excluding certain combinations a priori based on cluster properties. Solution quality could be improved by implementing one of the many existing improvement heuristics such as insertion and route swapping, once we have a set of initial solutions. Also, once we have a feasible solution, we could incrementally add relevant routes to the route set R and send the problem back to CPLEX to attempt to find an improved solution. This last idea might provide some insight into the sub-problems of an eventual column generation approach.

An exact Column Generation (CG) approach could be explored as well. Given that we have an initial solution that gives us a reasonable upper bound on the objective function, it is reasonable to envision the existence of a sub-problem that could generate sets of additional routes to consider. Given the structure of the problem in DR Congo and in particular the sparse nature of the hospital nodes, we anticipate that a CG approach could find an optimal solution relatively quickly because as the algorithm progresses, the reduced dual-costs will prevent a large proportion of potential routes from being generated and evaluated.

Improvements to implementation

The current implementation was written in Python for rapid development and testing and its interoperability with third party components such as the Postgres database and CPLEX. Currently, the route generation process accounts for about 50% of the processing time. CPLEX solves the resulting problem and generates the solution pool in about one second, whereas the route generation process takes about 30 seconds. Performance improvements could be achieved by extracting the route generation portion to C++ and employing multi-threaded features. Moreover, once the initial problem has been solved once, a better re-use of existing TSP calculations could increase performance even further for subsequent problem execution with different parameters. Finally, open source optimizers could be explored as alternatives to CPLEX in order to improve accessibility to humanitarian organizations operating on limited budgets.

Improvements and extensions to the web-based visualization tool

The current web-based visualization tool is a prototype. There are numerous potential performance improvements that could be achieved via back-end data caching and front-end processing of the geospatial data. Also, there are numerous extensions to the tool that could be explored, including visualization controls to toggle visibility of certain routing elements and animation to improve understanding of the solutions. Finally, extending the tool to allow administrators to execute new problems with different input parameters such as custom client filtering and on-site visit requirements would increase usability substantially. The technical knowledge required to install, configure, and execute the model could be wrapped in a friendly user interface and be accessed from a tablet or smart phone.

Commercial applications

Although a humanitarian application initially motivated the CVRP-SyncPD formulation and heuristic, commercial applications might also exist. Specifically, in the place of a supervisor we might consider an expensive light-weight asset (such as an expensive machine) that is needed at a set of locations that also require material deliveries but at different frequencies. Combining the machining activity (instead of a supervisor visit) with existing delivery routes might help reduce overall cost.

Assessing applicability based on cluster density

As we were testing our implementation with different input parameters, we noticed that if the average demand was higher, the maximum cluster size (evidently) got smaller. When we had medication cluster sizes of three nodes instead of four nodes, the results were even more promising, offering monthly deliveries and supervision at slightly less cost than the min-coord strategy. In general, we suspect that two factors may contribute to additional savings: increased density of hospital nodes and reduced number of nodes per medication cluster. This is related to the cost savings that occur by sharing the return segment of routes back to the depot: when the total distance between hospitals in a medication cluster increases (by way of node sparseness or increased number of nodes in a cluster) the savings of sharing the return segment diminish. Although we cannot change the density of the hospitals for our problem, if we see a rise in demand for medication supplies, we can anticipate additional savings with a synchronized solution. Our experience indicates that with a stable and reasonably-priced delivery and supervision schedule, medication prices will decrease, leading to an increase in demand. Moreover, when assessing whether the CVRP-SyncPD formulation and heuristic might be applicable in other contexts, the relative density of the nodes could be an indicator to consider.

6.3 Conclusion

In this thesis we have presented the Capacitated Vehicle Routing Problem with Synchronized Pick-ups and Drop-offs, a novel routing model for a synchronized CVRP. We have shown that in the case of Bandundu, a synchronized routing strategy would allow hospital administrators to implement a monthly delivery schedule at a cost only slightly higher than with the current four-month delivery schedule. Given the isolated contexts in which administrators in rural hospitals find themselves, and the high cost (financial cost and opportunity cost) of sending supervisors out to equip and empower local administrators sustainably, a quick discussion with the supervisor on the airstrip as medications are being unloaded is simply insufficient. Despite the cost, the two to three day onsite supervisor visit is a key factor to sustainably improving pharmacy management and equipping and empowering local administrators. The results of this thesis illustrate to a sufficient degree that the option of monthly medication delivery with synchronized supervision is feasible, and if applied appropriately, can help coordinators in their goal to improve healthcare quality and accessibility to rural populations.

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Appendix I

Input datasets

In this section we describe how we obtained and constructed the input datasets required for our implementation. Specifically, we outline the source of the following inputs: hospitals, depot location, airstrip locations, and vehicle specifications.

Hospitals

Estimating hospital locations

None of the coordinates for the general reference hospitals in the DR Congo were available at the time of the research. Hospital coordinates were estimated by merging two geospatial datasets: DR Congo health zone shapes and place names. The delimitations of the health zones were originally collected by the DR Congo's Ministry of Health in 2003 [28] and subsequently compiled into digital geometry by IMA World Health [20] in 2006. Coordinates and names for populated areas of the DR Congo were obtained from the GeoNames website [18].

The procedure used for estimating the hospital locations is as follows: For each health zone we selected name spaces (e.g. city/village) if the name space was located inside the health zone boundaries. Then, for each health zone we iterated over its named spaces and if there was a single exact match between a name space and its associated health zone, we associated the coordinates of that name space with the general reference hospital. In other words, if there was a single city/village with the same name as the health zone and that city/village was located inside the health zone, we made an educated guess that the hospital was situated in that locality. Unfortunately, there was not always an exact one-to-one match between health zone names and place names. Consequently, we used the matching logic outlined in Table I.I to ensure that we had a single hospital assigned to each health zone. [1]

Match type	Match description
Exact match	A single name space located within the health zone matched the health zone name exactly
Pair10km	If two name space located within the health zone matched the health zone name exactly and were less than 10 km apart, the coordinates of one name space were chosen at random.
ClosestToCentroid	If multiple name spaces located within the health zone matched the health zone name exactly and were >10 km apart we chose the one closest to centroid of the Health Zone
Centroid	If no name space located within the health zone matched the health zone name we used the centroid of the health zone as a estimate for the location of the hospital

Table I.I: Rules for estimating hospital locations

Hospital inclusion criteria

When considering which hospitals would most benefit from a coordinated delivery and supervision plan, we choose to exclude hospitals that were close to the depot location (Kikwit) because travel to these locations is relatively inexpensive, reducing the benefit from coordinated air transport. Consequently, we focused on 41 hospitals that were far enough away from the depot to warrant attention. In the implementation we arbitrarily chose hospitals based on a minimum of 100 km distance from the depot as a threshold, but we could also exclude hospitals close to major highways because they can easily obtain medications via ground vehicles. These kinds of geospatial filters can easily be set up as additional input parameters. See Appendix III for a complete list of hospitals included in this case study. Figure I.1 displays a map of included and excluded hospitals. In the model, we refer to the index set of included hospitals as $V = \{1,...,41\}$.

The resulting matrix of the estimated hospital coordinates was an input for the model. We used this dataset to determine the distance matrix between nodes and eventually to generate an adjacency matrix. The results of the merging process for hospitals in the

1. The task of preparing the shapes and merging the raw datasets together into Postgres tables was commissioned by the author and completed by Hans Fast.

province of Bandundu can be found in Appendix II. Figure I.1 shows a map of the estimated hospital locations.

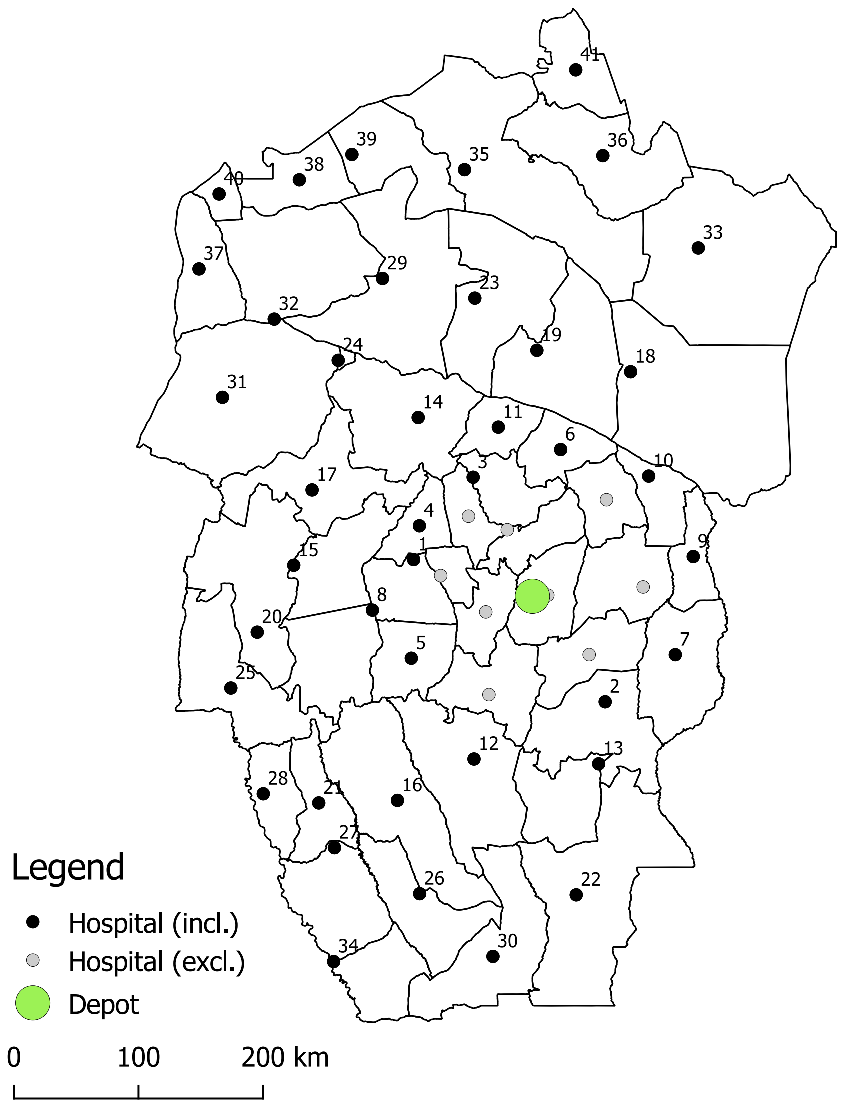

Figure I.1: Approximate locations of hospitals and depot in the province of Bandundu

Monthly hospital demand

Medication demand data for all hospitals in the province of Bandundu is unavailable. Consequently, we estimated the average monthly demand to be approximately 240 kgs, based on our experience as a project coordinator with four hospitals located in or near

Bandundu province (HGR Kajiji, HGR Mukedi, HGR Nyanga, HS Kikwit). In order to simulate a realistic level of variance between hospitals, we generated average monthly demands for each hospital using a Gaussian distribution with $\mu = 240$ and $\sigma = 20$. See the demand vector in Appendix IV. In the model, we refer to the hospital demand vector as w.

Depot location

There were several potential depot sites. We chose the city of Kikwit for the purpose of this study. The main reason for this choice was Kikwit's central location, proximity to the national highway, numerous local pharmaceutical companies, and airstrip facilities. Note that in practice the 'depot' is not a single storage unit per se, but a combination of private and public suppliers from which we can make monthly orders with an acceptable service level. The choice of using local pharmaceutical distributors rather than importing medications directly from foreign (typically Western) pharma companies is based on the desire to support local businesses in light of encouraging a more sustainable distribution chain.

Airstrips

Airstrip locations, specifications and conditions for the DR Congo were obtained from MAF [25] [2]. Many general reference hospitals have sand or grass airstrips that can be maintained locally. Figure I.2 shows that currently only 15 out 41 locations actually have an airstrip within 5 km of the hospital. For the purposes of this research, we assume that hospitals have a suitable airstrip at the same location. Although this is not realistic, we felt that using the estimated hospital locations was significantly more realistic than

2. A well-known and respected international not-for-profit aviation organization, Missionary Aviation Fellowship (MAF), provides aviation and technical assistance to various groups in the DR Congo including humanitarian NGOs. MAF offers these services well below actual cost. Pilots, mechanics, and flight coordinators volunteer its time. The pricing aims to cover operational costs such as fuel and local aviation licensing. Financing for new aircraft is obtained through fundraising. MAF graciously supplied pricing information and specifications for each of their aircraft in the DR Congo to assist this research.

generating a completely random set of points. Moreover, airstrips could eventually be added to accommodate a province-wide.

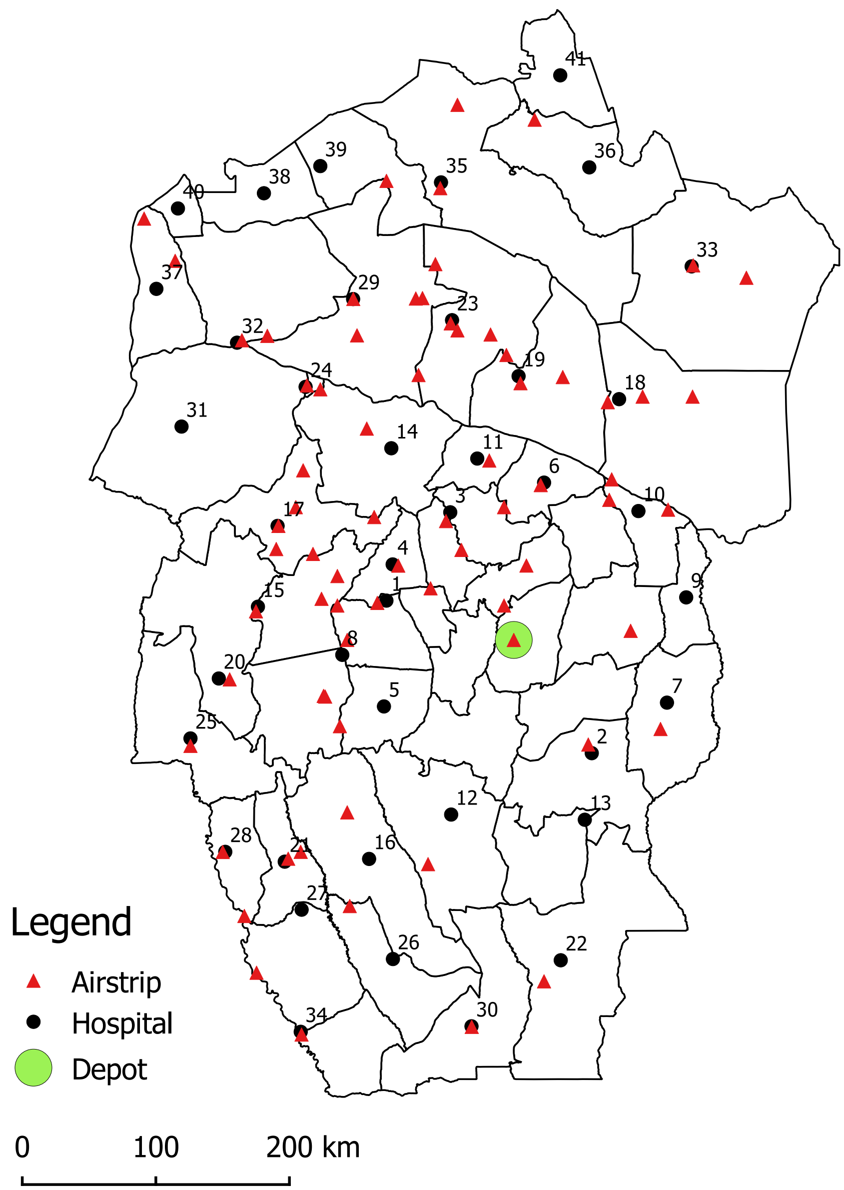

Figure I.2: Airstrips in the province of Bandundu, DR Congo
Coordinates were provided by MAF. The map was generated in QGIS by the author.

Aircraft specifications

There are several air carriers in the DR Congo, but we chose to use specifications from MAF aircraft because we had experience using their services and the data was readily available. The available aircraft that we considered in this study are detailed in Table I.II [3] but could easily be expanded to include other aircraft. The actual cost/km depends on a number of factors such as fuel price and client stratification. The smallest aircraft, the Cessna 182, is a very small aircraft used to transport a supervisor and maximum 50 kg of medications. This aircraft is chosen to bring back a supervisor at the end of the month with no medication delivery requirement. The Cessna 206 can take a supervisor and a maximum of 300 kg of medications. The Cessna 209, also known as the Caravan, can handle up to 1000 kg of medications and a supervisor.

Index h	Aircraft	Rate: USD/km	Capacity (kg)
0	Cessna 182	2.95	50
1	Cessna 206	4.00	300
2	Cessna 209	6.10	1000

Table I.II: Specifications of aircraft used in this study (as of 2012). Aircraft name, cost per kilometre, and maximum capacity for a single trip

Pilots have additional restrictions to consider when determining feasibility of a route candidate. In particular, load/balance and weight of fuel need to be considered. For reasons of scope, we did not include these factors in the implementation, but they could be added if necessary. Moreover, it is assumed that most routes proposed in the implementation (if not all) would meet the requirements.

3. The prices listed in this table were the lowest pricing available to some clients in 2012 and do not necessarily reflect the current pricing or specifications.

Appendix II

Hospital location merging results

Node index j	**Health zone name**	**Match type**
1	Masi-Manimba	pair10km
2	Gungu	closest to centroid
3	Djuma	single match
4	Yasa-Bonga	centroid
5	Moanza	centroid
6	Mokala	centroid
7	Mukedi	closest to centroid
8	Kimbao	single match
9	Koshibanda	centroid
10	Ipamu	single match
11	Sia	centroid
12	Feshi	closest to centroid
13	Kisandji	single match
14	Bagata	pair10km
15	Kenge	single match
16	Mwela-Lembwa	centroid
17	Kikongo	single match
18	Oshwe	pair10km
19	Bosobe	single match
20	Boko	pair10km
21	Wamba-Lwadi	centroid
22	Kahemba	closest to centroid
23	Bokoro	single match
24	Bandundu	pair10km
25	Popokabaka	pair10km
26	Panzi	single match
27	Kitenda	single match
28	Kasongo-Lunda	closest to centroid
29	Nioki	single match
30	Kajiji	closest to centroid
31	Kwamouth	closest to centroid
32	Mushie	pair10km
33	Mimia	single match
34	Tembo	pair10km
35	Inongo	closest to centroid
36	Kiri	centroid
37	Bolobo	closest to centroid
38	Ntandembelo	centroid
39	Bandjow-Moke	single match
40	Yumbi	closest to centroid
41	Pendjwa	centroid

Appendix III

Estimated hospital and depot locations

Table III.I: Estimated hospital and depot locations in longitude and latitude for hospitals included in the case study

Node index j	**Health zone name**	**Longitude**	**Latitude**
0	DEPOT (Kikwit)	18.82407	-5.02130
1	Masi-Manimba	17.91667	-4.76667
2	Gungu	19.31407	-5.79978
3	Djuma	18.35102	-4.16783
4	Yasa-Bonga	17.95905	-4.52124
5	Moanza	17.89981	-5.48345
6	Mokala	18.98946	-3.96648
7	Mukedi	19.82568	-5.45779
8	Kimbao	17.61633	-5.13415
9	Koshibanda	19.95664	-4.74466
10	Ipamu	19.63151	-4.15947
11	Sia	18.53477	-3.80327
12	Feshi	18.35722	-6.21495
13	Kisandji	19.26667	-6.25000
14	Bagata	17.95000	-3.73333
15	Kenge	17.04237	-4.80799
16	Mwela-Lembwa	17.79878	-6.51475
17	Kikongo	17.17609	-4.26021
18	Oshwe	19.50000	-3.40000
19	Bosobe	18.81599	-3.24386
20	Boko	16.77602	-5.29424
21	Wamba-Lwadi	17.22470	-6.53337
22	Kahemba	19.10292	-7.19913
23	Bokoro	18.36326	-2.86384
24	Bandundu	17.36667	-3.31667
25	Popokabaka	16.58333	-5.70000
26	Panzi	17.96091	-7.19076
27	Kitenda	17.33952	-6.85776
28	Kasongo-Lunda	16.82000	-6.46700
29	Nioki	17.69001	-2.72037
30	Kajiji	18.49558	-7.64447
31	Kwamouth	16.52243	-3.58621
32	Mushie	16.90000	-3.01667
33	Mimia	19.99368	-2.49828
34	Tembo	17.33351	-7.68156
35	Inongo	18.28810	-1.92750
36	Kiri	19.29720	-1.82533
37	Bolobo	16.35123	-2.65093
38	Ntandembelo	17.08311	-2.00133
39	Bandjow-Moke	17.46667	-1.81667
40	Yumbi	16.49800	-2.10467
41	Pendjwa	19.09955	-1.19966

Appendix IV

Monthly hospital medication demand

Table IV.I: Monthly hospital medication demand generated randomly from a Gaussian distribution based on average demands from sample hospitals.

Node index j	**Demand (kg)**	**Demand (kg)**
1	Masi-Manimba	275
2	Gungu	248
3	Djuma	260
4	Yasa-Bonga	285
5	Moanza	277
6	Mokala	221
7	Mukedi	259
8	Kimbao	237
9	Koshibanda	238
10	Ipamu	248
11	Sia	243
12	Feshi	269
13	Kisandji	255
14	Bagata	242
15	Kenge	249
16	Mwela-Lembwa	247
17	Kikongo	270
18	Oshwe	236
19	Bosobe	246
20	Boko	223
21	Wamba-Lwadi	189
22	Kahemba	253
23	Bokoro	257
24	Bandundu	225
25	Popokabaka	285
26	Panzi	211
27	Kitenda	241
28	Kasongo-Lunda	236
29	Nioki	271
30	Kajiji	269
31	Kwamouth	243
32	Mushie	248
33	Mimia	222
34	Tembo	200
35	Inongo	233
36	Kiri	243
37	Bolobo	265
38	Ntandembelo	264
39	Bandjow-Moke	232
40	Yumbi	234
41	Pendjwa	219

Appendix V

Distances to depot and costs of return flights

Table V.I: Distances to depot and costs of return flights for each hospital

Hospital index	Hospital name	Distance to depot (km)	Cost of return flight with Cessna 209 ($6.1/km)	Cost of return flight with Cessna 182 ($2.95/km)
1	Masi-Manimba	104	1274	616
2	Gungu	102	1246	603
3	Djuma	108	1323	640
4	Yasa-Bonga	111	1352	654
5	Moanza	115	1397	676
6	Mokala	119	1448	700
7	Mukedi	121	1477	714
8	Kimbao	134	1639	793
9	Koshibanda	129	1576	762
10	Ipamu	131	1600	774
11	Sia	139	1698	821
12	Feshi	142	1738	840
13	Kisandji	145	1771	856
14	Bagata	173	2110	1020
15	Kenge	199	2425	1173
16	Mwela-Lembwa	201	2453	1186
17	Kikongo	201	2456	1188
18	Oshwe	195	2382	1152
19	Bosobe	198	2411	1166
20	Boko	229	2792	1350
21	Wamba-Lwadi	244	2978	1440
22	Kahemba	244	2978	1440
23	Bokoro	245	2992	1447
24	Bandundu	249	3039	1470
25	Popokabaka	259	3163	1530
26	Panzi	259	3165	1531
27	Kitenda	262	3197	1546
28	Kasongo-Lunda	274	3341	1616
29	Nioki	285	3478	1682
30	Kajiji	294	3586	1734
31	Kwamouth	301	3672	1776
32	Mushie	309	3765	1821
33	Mimia	309	3771	1824
34	Tembo	339	4131	1998
35	Inongo	349	4259	2060
36	Kiri	359	4383	2119
37	Bolobo	380	4641	2245
38	Ntandembelo	387	4727	2286
39	Bandjow-Moke	387	4720	2283
40	Yumbi	414	5057	2445
41	Pendjwa	426	5198	2514
	Total	9574	116808	56489
	Average	234	2849	1378